

PREPARED FOR SUBMISSION TO JHEP

# Neutralino-Neutralino pair annihilation crosssections with flavour violation

---

**V. Suryanarayana Mummidi**

*Centre for High Energy Physics, Indian Institute of Science, Bangalore 560012*

*E-mail:* [soori9@cts.iisc.ernet.in](mailto:soori9@cts.iisc.ernet.in)

**ABSTRACT:** We present the first exact and most general neutralino-neutralino annihilation cross sections in the presence of flavor violation. The existing set of calculations in literature do not include flavour violation. These calculations are important for the calculation of relic density and indirect detection of dark matter in various experiments.

---

## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Thermal relic density</b>	<b>4</b>
<b>3</b>	<b>Cross sections</b>	<b>6</b>
3.1	$\chi_i^0 \chi_j^0 \rightarrow H_m^0 H_n^0$	6
3.2	$\chi_i^0 \chi_j^0 \rightarrow A_1^0 A_1^0$	7
3.3	$\chi_i^0 \chi_j^0 \rightarrow H_1^0 A_1^0$	9
3.4	$\chi_i^0 \chi_j^0 \rightarrow H_2^0 A_1^0$	11
3.5	$\chi_i^0 \chi_j^0 \rightarrow H^+ H^-$	12
3.6	$\chi_i^0 \chi_j^0 \rightarrow W^\pm H^\mp$	14
3.7	$\chi_i^0 \chi_j^0 \rightarrow Z H_n^0$	16
3.8	$\chi_i^0 \chi_j^0 \rightarrow Z H_n^0$	18
3.9	$\chi_i^0 \chi_j^0 \rightarrow W^\pm W^\mp$	20
3.10	$\chi_i^0 \chi_j^0 \rightarrow Z Z$	22
3.11	$\chi_i^0 \chi_j^0 \rightarrow f_m \bar{f}_n$	23
<b>4</b>	<b>Summary</b>	<b>29</b>
<b>A</b>	<b>Mixing matrices and Coupling constants</b>	<b>29</b>
<b>B</b>	<b>Auxiliary functions</b>	<b>33</b>

---

## 1 Introduction

One of the unresolved issues of the twentieth century physics is the nature of dark matter. As per the present knowledge on the content of the Universe, major part of the Universe is not made of protons and neutrons but by a new form of matter called dark matter. It is six times more abundant than the matter in the visible Universe.

There is sufficient evidence suggesting the existence of dark matter. And it does not fall into the category of any known matter. One of the well studied dark matter candidates are Weakly Interacting Massive Particles(WIMPs). WIMPs are electrically neutral, stable and they interact through weak and gravitational interactions. There is no particle, in Standard Model (SM), which could act as dark matter candidate except for neutrino. As neutrinos are stable, weakly interacting and electrically neutral they are suitable candidate for dark matter. However, neutrinos being relativistic particles they could only contribute to the hot dark matter. But to explain the structure formation in the Universe dark matter has to be cold. One of the popular extensions of SM which has darkmatter candidate is Minimally Supersymmetric Standard Model (MSSM)[1]. An attractive feature of supersymmetric models

is the existence of quantum number called R-symmetry. Under this symmetry non supersymmetric particle has R quantum number +1 while their super partners are assigned -1. As all the supersymmetric interactions should obey R-symmetry, supersymmetric particles are produced and annihilated in pairs. This makes lightest supersymmetric particles stable. If lightest supersymmetric particle were neutral it could be a dark matter candidate. In R-parity conserving supersymmetric extensions of the Standard Model there must exist a LSP which could be a dark matter candidate. They exist in the early Universe along with other particles and eventually annihilate and coannihilate into Standard Model particles. As the Universe expands some of the LSPs will cease to interact and will be left over in the present Universe as thermal relics. More detailed calculation of thermal relic density of dark matter is presented in the next section.

In MSSM, owing to the fact that neutral gauginos bino, wino and Higgsino have identical weak isospin quantum number, they mix to form four mass eigen states called Neutralinos. They are denoted by  $\chi_1^0, \chi_2^0, \chi_3^0$  and  $\chi_4^0$ . If R-parity is conserved, lightest neutralino is the LSP in most of the models. It is compelling to consider neutralinos in MSSM as the dark matter candidate. In the mixed basis,  $\psi^T = (\tilde{B}, \tilde{W}, \tilde{H}_1, \tilde{H}_2)$ , the neutralino part of the Lagrangian is

$$\mathcal{L}_{neutralino} = -\frac{1}{2} \psi^T M_\chi \psi \quad (1.1)$$

Neutralino mass matrix  $M_\chi$  is given by

$$\begin{pmatrix} M_1 & 0 & -\frac{e v_1}{2 c_W} & \frac{e v_2}{2 c_W} \\ 0 & M_2 & \frac{e v_1}{2 s_W} & -\frac{e v_2}{2 s_W} \\ -\frac{e v_1}{2 c_W} & \frac{e v_1}{2 s_W} & 0 & -\mu \\ \frac{e v_2}{2 c_W} & -\frac{e v_2}{2 s_W} & -\mu & 0 \end{pmatrix} \quad (1.2)$$

where  $M_1$  and  $M_2$  are masses of neutral gauginos Bino and Wino respectively.  $\mu$  is the Higgsino mass parameter, 'e' is the gauge coupling constant,  $v_1$  and  $v_2$  are vacuum expectation values and  $\theta_W$  ( $c_W = \cos \theta_W$  and  $s_W = \sin \theta_W$ ) is the Weinberg angle. Mass eigen states can be obtained by diagonalizing the neutralino mass matrix  $M_\chi$  by a unitary matrix  $Z_N$ :

$$\chi_i^0 = (Z_N)_{ij} \psi_j \quad (1.3)$$

so that

$$Z_N^{-1} M_\chi Z_N = \begin{pmatrix} m_{\chi_1^0} & 0 & 0 & 0 \\ 0 & m_{\chi_2^0} & 0 & 0 \\ 0 & 0 & m_{\chi_3^0} & 0 \\ 0 & 0 & 0 & m_{\chi_4^0} \end{pmatrix} \quad (1.4)$$

With high precision measurements of dark matter relic density, it may not be of distant possibility to constrain supersymmetric models. If heavier supersymmetric states predicted by supersymmetry are discovered at LHC, one can study the possibility that neutralino is

the dark matter particle. In this context knowing the thermal relic density precisely has utmost importance. In standard cosmology thermal relic density of dark matter is given by

$$\Omega_\chi = \frac{n_\chi m_\chi}{\rho_c} \quad (1.5)$$

where  $\rho_c$  is the critical density given by  $\frac{3H^2}{8\pi G}$ .  $G$  and  $H$  are gravitational constant and present day Hubble parameter respectively. The solution of Boltzmann equation gives the number density of any particle in the Early Universe as Universe evolves. We have assumed that dark matter is thermally produced and is in thermal equilibrium with the background in the early phase of the Universe, its number density  $n_\chi$  can be calculated by solving Boltzmann rate equation

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma|v|\rangle(n_\chi^2 - n_{\chi,eq}^2) \quad (1.6)$$

Where  $t$  is time, and  $\langle\sigma_{X\bar{X}}|v|\rangle$  is the thermally averaged cross section times the relative velocity. One can solve eq. (1.6) to obtain the number density at  $t=0$  (today) provided,  $\langle\sigma|v|\rangle$  is known. As it was pointed out in [2], if the masses of the LSP and next to lightest supersymmetric particle (nLSP) are degenerate coannihilations between LSP and nLSP prominently alters the number density. So a general dark matter density calculation should include coannihilation of neutralino(LSP) with nLSP. In MSSM, nLSP could be neutralino, chargino or a slepton depending on the boundary conditions at the supersymmetry breaking scale<sup>1</sup>. Therefore, the general calculation of neutralino number density involves consideration of following cross sections:

- |  |   |
|--|---|
| (i) Neutralino (LSP)-Neutralino(LSP)   | (ii) Neutralino (LSP)-Neutralino (nLSP) |
| (iii) Neutralino (LSP)-Chargino (nLSP) | (iv) Neutralino (LSP)- Slepton (nLSP)   |

So the problem of calculation of today's neutralino dark matter relic density is usually the calculation of the cross sections with the initial states given in above cases (i), (ii), (iii) and (iv) into all standard model particles. The relic density of neutralinos in the MSSM has been calculated by several authors over the years [2–12]. Nihei et al. [9, 10] have considered Neutralino cross sections extensively except for the case (ii). They also have assumed neutralino mixing matrix  $Z_N$  to be real for simplicity and not included flavour violation in the supersymmetric soft sector. Authors in [11, 12] have considered effect of flavor violation on coannihilation and annihilations. In most calculations of the cross sections flavor violation is not considered seriously because of the existing low energy constraints from rare decays like  $\mu \rightarrow e\gamma$ ,  $\tau \rightarrow \mu\gamma$ , etc. in the leptonic sector and  $b \rightarrow s\gamma$ ,  $B - \bar{B}^0$ ,  $B_s - \bar{B}_s$ , in the hadronic sector. However, it is known that there are regions in the parameter space where these constraints are highly weakened. In the leptonic sector cancellations of amplitude of  $l_j \rightarrow l_i\gamma$  are widely noted [13]. These parameter regions increase if one considers a more general SUSY breaking framework like the one with the non-universal gauge masses [14, 15]. In the hadronic sector, constraints are weakened typically for a relatively heavy squark spectrum  $\sim 1$  TeV or so. In fact, so much so that, in mini-split models one expects a little bit of flavor violation to be present in the 2-3 sector [16, 17].

---

<sup>1</sup>Stops or gluinos could also be nLSP. One has to do case by case study of cross sections

Process	Exchanged Particle	
	s-channel	t- and u-channel
$\chi_i^0 \chi_j^0 \rightarrow H_m^0 H_n^0$	$H_k^0$	$\chi_k^0$
$\chi_i^0 \chi_j^0 \rightarrow A_1^0 A_1^0$	$H_k^0$	$\chi_k^0$
$\chi_i^0 \chi_j^0 \rightarrow H_1^0 A_1^0$	$A_1^0, Z$	$\chi_k^0$
$\chi_i^0 \chi_j^0 \rightarrow H_2^0 A_1^0$	$H_k^0$	$\chi_k^0$
$\chi_i^0 \chi_j^0 \rightarrow H_1^+ H_1^-$	$H_k^0, Z$	$\chi_k^\pm$
$\chi_i^0 \chi_j^0 \rightarrow W^\pm H_1^\mp$	$H_k^0, A_1^0$	$\chi_k^\pm$
$\chi_i^0 \chi_j^0 \rightarrow Z H_n^0$	$Z, A_1^0$	$\chi_k^0$
$\chi_i^0 \chi_j^0 \rightarrow Z A_1^0$	$H_k^0$	$\chi_k^0$
$\chi_i^0 \chi_j^0 \rightarrow W^+ W^-$	$H_k^0, Z$	$\chi_k^\pm$
$\chi_i^0 \chi_j^0 \rightarrow Z Z$	$H_k^0$	$\chi_k^0$
$\chi_i^0 \chi_j^0 \rightarrow f_m f_n$	$H_k^0, A_1^0, Z$	$\tilde{f}_k$

**Table 1.** A complete set of neutralino pair-annihilation channels into tree-level two-body final states in the MSSM. The index k runs as follows: For neutralinos k=1,...,4; for charginos k=1,2; for sfermions k=1,...,6; for Higgs k=1,2 and for fermions m,n=1,...,6

The purpose of our paper is to present most general neutralino(LSP)-neutralino (LSP/nLSP) cross sections without assuming minimal flavor violation or real mixing matrix for neutral gauge eigen state matrix  $Z_N$ . And we present case (iii) and (iv) in our future work. We hope to extend this analysis Sommerfeld enhancement phenomena in a future work. It should be noted that these cross sections are also useful to study indirect detection of dark matter. With flavor becoming important in the future experiments, the large presence of flavor violation in the supersymmetric soft sector could possibly be probed by indirect experiments.

The neutralino pair annihilation and coannihilation cross section with neutralino in this paper is organized and presented as follows. In table.1, we present the various possible SM final states. For each set of final states, we present the channels and the corresponding mediators. In section.2, we sketch the calculation of thermal relic density. In section.3, we present cross sections corresponding to final states presented in table.1.

## 2 Thermal relic density

In this section, we outline the calculation of relic density and write it as one dimensional integral [2, 8, 10].

Consider annihilation of N supersymmetric particles  $\chi_i$  (i=1,2,...,N-1, N). And their mass and number density are given by  $m_i$  and  $n_i$  respectively. The decay rate of supersymmetric particles  $\chi_i$  which are heavier than the LSP is much faster than the age of the Universe. In the case of R-parity conserved models, like the one we have considered here, these particles decay into the LSP. If LSP were the dark matter particle, its final abundance

is described by the sum of the number density of all the supersymmetric particles:

$$n = \sum_i^N n_i \quad (2.1)$$

For n, Boltzmann evolution equation is given by [8]

$$\frac{dn}{dt} + 3Hn = - \sum_{i,j}^N \langle \sigma_{ij} v_{ij} \rangle (n_i n_j - n_i^{eq} n_j^{eq}) \quad (2.2)$$

where

$$\sigma_{ij} = \sigma(\chi_i \chi_j \rightarrow \text{All}) \quad (2.3)$$

Taking into the fact that supersymmetric particles are non relativistic and their number density distributions are suppressed by a Boltzmann factor. In this scenario, thermal distribution of density  $n_i$  remain in equilibrium. So the ratio  $\frac{n_i}{n}$  is equal to the equilibrium value:

$$\frac{n_i}{n} \sim \frac{n_i^{eq}}{n^{eq}} \quad (2.4)$$

then the eq.2.2 is simplified to

$$\frac{dn}{dt} + 3Hn = - \sum_{i,j}^N \langle \sigma_{eff} v \rangle (n^2 - n_{eq}^2) \quad (2.5)$$

where

$$\langle \sigma_{eff} v \rangle = \sum_{i,j}^N \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{eq}}{n^{eq}} \frac{n_j^{eq}}{n^{eq}} \quad (2.6)$$

By using Maxwell-Boltzmann equilibrium distribution the thermal average  $\langle \sigma_{ij} v_{ij} \rangle$  is written as

$$\langle \sigma_{ij} v_{ij} \rangle = \frac{\int d^3 p_i d^3 p_j \sigma v_{ij} e^{-E_i/T} e^{-E_j/T}}{\int d^3 p_i d^3 p_j e^{-E_i/T} e^{-E_j/T}} \quad (2.7)$$

where  $p_i = (E_i, \mathbf{p}_i)$  have usual meaning; energy and momenta of the two annihilating particles and T is the equilibrium temperature. Eq.2.6 can be rewritten as

$$\langle \sigma_{eff} v \rangle = \frac{\int_{4m_1^2}^{\infty} ds s^{3/2} K_1 \left( \frac{\sqrt{s}}{T} \right) \sum_{ij}^N \beta_f^2(s, m_i^2, m_j^2) \frac{g_i g_j}{g_1^2} \sigma_{ij}(s)}{8 m_1^4 T \left[ \sum_i^N \frac{g_i}{g_1} \frac{m_i^2}{m_1^2} K_2 \left( \frac{m_i}{T} \right) \right]^2} \quad (2.8)$$

where  $g_i$  is number of degrees of freedom,  $K_i$  is modified Bessel function of order 'i' and  $\beta_f(s, m_i, m_j)$  is the kinetic function defined as

$$\beta_f(s, m_i, m_j) = \frac{\sqrt{s - (m_i^2 + m_j^2)} \sqrt{s - (m_i^2 - m_j^2)}}{s} \quad (2.9)$$

### 3 Cross sections

In this section, general cross section for the cases (i) and (ii) mentioned in the introduction is given. To ease the process of comparison of our results with the ones already present in the literature[10], we rewrite  $\sigma_{ij}(s)$  as

$$\sigma_{ij}(s) = \frac{s}{16\pi \beta_f(s, m_i, m_j)} \sum_{\phi_1 \phi_2} [c \theta(s - (m_1^2 + m_2^2)) \beta_f(s, m_1, m_2) \tilde{w}_{\phi_1 \phi_2}(s)] \quad (3.1)$$

where  $m_1$  and  $m_2$  are the masses of the final states  $\phi_1$  and  $\phi_2$  respectively. And ‘c’ is the colour factor. If  $\chi_i^0 \chi_j^0 \rightarrow \phi_1 \phi_2$ , then the function  $\tilde{w}(s)_{\phi_1 \phi_2}$ , in the centre of mass frame, given as

$$\tilde{w}(s)_{\phi_1 \phi_2} \equiv \frac{1}{2} \int_{-1}^1 d \cos \theta_{CM} |\mathcal{A}(\chi_i^0 \chi_j^0 \rightarrow \phi_1 \phi_2)|^2 \quad (3.2)$$

where  $\theta_{CM}$  denotes the scattering. We will follow Table.1 in presenting explicit expressions for  $\tilde{w}_{\phi_1 \phi_2}(s)$  for all the two-body final states.

#### 3.1 $\chi_i^0 \chi_j^0 \rightarrow H_m^0 H_n^0$

This process is the sum of the s-channel diagram through CP even Higgs boson ( $H_1^0$  and  $H_2^0$ ) and the t- and u-channel neutralino ( $\chi_k^0$ ,  $k = 1, \dots, 4$ ) exchange. The resulting cross section, including the s, t, u and possible interference processes among themselves, is given by.

$$\tilde{w}_{H_m^0, H_n^0} = \sum_{k,l=1,2} \tilde{w}_{H_m^0, H_n^0}^{(H_k^0, H_l^0)} + \sum_{k=1,2} \sum_{l=1,4} \tilde{w}_{H_m^0, H_n^0}^{(H_k^0, \chi_l^0)} + \sum_{k,l=1,4} \tilde{w}_{H_m^0, H_n^0}^{(\chi_k^0, \chi_l^0)} \quad (3.3)$$

- CP-even Higgs boson ( $H_k^0, H_l^0$ ) exchange:

$$\tilde{w}_{H_m^0, H_n^0}^{(H_k^0, H_l^0)} = \frac{1}{2} \frac{C_{H_k^0 H_m^0 H_n^0} C_{H_l^0 H_m^0 H_n^0}^*}{2(s - m_{H_k^0}^2)(s - m_{H_l^0}^2)} \left( f_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)}(s - m_i^2 - m_j^2) - 2 m_i m_j g_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} \right) \quad (3.4)$$

- Neutralino ( $\chi_k^0 - \chi_l^0$ ) exchange:

$$\begin{aligned} \tilde{w}_{H_m^0, H_n^0}^{t(\chi_k^0, \chi_l^0)} &= 4 d m_{\chi_k^0} m_{\chi_l^0} \Gamma_{H_m^0 H_n^0}^{1(\chi_k^0 - \chi_l^0)} + \beta_1(H_m^0, H_n^0, \chi_k^0, \chi_l^0) \Gamma_{H_m^0 H_n^0}^{2(\chi_k^0 - \chi_l^0)} \\ &\quad - 4 m_i m_j m_{\chi_k^0} m_{\chi_l^0} \Gamma_{H_m^0 H_n^0}^{3(\chi_k^0 - \chi_l^0)} - m_i m_j \beta_4(H_m^0, H_n^0, \chi_k^0, \chi_l^0) \Gamma_{H_m^0 H_n^0}^{4(\chi_k^0 - \chi_l^0)} \\ &\quad - m_j (m_{\chi_k^0} \Gamma_{H_m^0 H_n^0}^{5(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{H_m^0 H_n^0}^{6(\chi_k^0 - \chi_l^0)}) \beta_3(H_m^0, H_n^0, \chi_k^0, \chi_l^0) \\ &\quad + (4 d m_i - \frac{m_i \omega_{H_m^0 H_n^0}}{s}) (m_{\chi_k^0} \Gamma_{H_m^0 H_n^0}^{7(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{H_m^0 H_n^0}^{8(\chi_k^0 - \chi_l^0)}) \end{aligned} \quad (3.5)$$

$$\begin{aligned} \tilde{w}_{H_m^0, H_n^0}^{u(\chi_k^0, \chi_l^0)} &= 4 d m_{\chi_k^0} m_{\chi_l^0} \Gamma_{H_n^0 H_m^0}^{1(\chi_k^0 - \chi_l^0)} + \beta_1(H_n^0, H_m^0, \chi_k^0, \chi_l^0) \Gamma_{H_n^0 H_m^0}^{2(\chi_k^0 - \chi_l^0)} \\ &\quad - 4 m_i m_j m_{\chi_k^0} m_{\chi_l^0} \Gamma_{H_n^0 H_m^0}^{3(\chi_k^0 - \chi_l^0)} - m_i m_j \beta_2(H_n^0, H_m^0, \chi_k^0, \chi_l^0) \Gamma_{H_n^0 H_m^0}^{4(\chi_k^0 - \chi_l^0)} \\ &\quad - m_j (m_{\chi_k^0} \Gamma_{H_n^0 H_m^0}^{5(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{H_n^0 H_m^0}^{6(\chi_k^0 - \chi_l^0)}) \beta_3(H_n^0, H_m^0, \chi_k^0, \chi_l^0) \\ &\quad + (4 d m_i - \frac{m_i \omega_{H_n^0 H_m^0}}{s}) (m_{\chi_k^0} \Gamma_{H_n^0 H_m^0}^{7(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{H_n^0 H_m^0}^{8(\chi_k^0 - \chi_l^0)}) \end{aligned} \quad (3.6)$$

$$\begin{aligned}
\tilde{w}_{H_m^0, H_n^0}^{t-u(\chi_k^0, \chi_l^0)} &= d m_{\chi_k^0} m_{\chi_l^0} s^2 J_1(H_m^0, H_n^0, \chi_k^0, \chi_l^0) I_{H_m^0 H_n^0}^{1(\chi_k^0 - \chi_l^0)} + J_2(H_m^0, H_n^0, \chi_k^0, \chi_l^0) \\
&\quad I_{H_m^0 H_n^0}^{2(\chi_k^0 - \chi_l^0)} - m_i m_j m_{\chi_k^0} m_{\chi_l^0} s^2 J_1(H_m^0, H_n^0, \chi_k^0, \chi_l^0) I_{H_m^0 H_n^0}^{3(\chi_k^0 - \chi_l^0)} \\
&\quad + m_i m_j s^2 (l_i/2 - d_{H_m^0 H_n^0} - m_i^2) J_1(H_m^0, H_n^0, \chi_k^0, \chi_l^0) I_{H_m^0 H_n^0}^{4(\chi_k^0 - \chi_l^0)} \\
&\quad + \frac{1}{4} m_j m_{\chi_k^0} s J_3(H_m^0, H_n^0, \chi_k^0, \chi_l^0) I_{H_m^0 H_n^0}^{5(\chi_k^0 - \chi_l^0)} + \frac{1}{4} m_j m_{\chi_l^0} s \\
&\quad J_3(H_m^0, H_n^0, \chi_l^0, \chi_k^0) I_{H_m^0 H_n^0}^{6(\chi_k^0 - \chi_l^0)} - \frac{1}{4} m_i m_{\chi_k^0} s J_4(H_m^0, H_n^0, \chi_k^0, \chi_l^0) \\
&\quad I_{H_m^0 H_n^0}^{7(\chi_k^0 - \chi_l^0)} + \frac{1}{4} m_i m_{\chi_l^0} s J_5(H_m^0, H_n^0, \chi_k^0, \chi_l^0) I_{H_m^0 H_n^0}^{8(\chi_k^0 - \chi_l^0)} \quad (3.7)
\end{aligned}$$

$$\tilde{w}_{H_m^0, H_n^0}^{(\chi_k^0, \chi_l^0)} = \tilde{w}_{H_m^0, H_n^0}^{t(\chi_k^0, \chi_l^0)} + \tilde{w}_{H_m^0, H_n^0}^{u(\chi_k^0, \chi_l^0)} + \left( \tilde{w}_{H_m^0, H_n^0}^{t-u(\chi_k^0, \chi_l^0)} + c.c. \right) \quad (3.8)$$

- CP-even Higgs boson ( $H_k^0$ ) and Neutralino ( $\chi_l^0$ ) Interference:

$$\begin{aligned}
\tilde{w}_{H_m^0, H_n^0}^{st(H_k^0, \chi_l^0)} &= -\frac{1}{2} m_i m_j m_{\chi_l^0} s L(\chi_l^0, H_m^0, H_n^0) J_{H_m^0 H_n^0}^{(1)(H_k^0 - \chi_l^0)} \\
&\quad - \frac{m_j}{4} \left( b_{H_m^0 H_n^0} - (R_{H_m^0 \chi_l^0} - 4 m_i^2 s) \right) \mathcal{A}(\chi_l^0, H_m^0, H_n^0) J_{H_m^0 H_n^0}^{(2)(H_k^0 - \chi_l^0)} \\
&\quad + \frac{1}{2} d m_{\chi_l^0} s L(\chi_l^0, H_m^0, H_n^0) J_{H_m^0 H_n^0}^{(3)(H_k^0 - \chi_l^0)} - \frac{m_i}{4} \left( b_{H_m^0 H_n^0} - (R_{H_m^0 \chi_l^0} \right. \\
&\quad \left. + 4 d s - \tau_{H_m^0 H_n^0} - \omega_{H_m^0 H_n^0}) \mathcal{A}(\chi_l^0, H_m^0, H_n^0) \right) J_{H_m^0 H_n^0}^{(4)(H_k^0 - \chi_l^0)} \quad (3.9)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{H_m^0, H_n^0}^{su(H_k^0, \chi_l^0)} &= -\frac{1}{2} m_i m_j m_{\chi_l^0} s L(\chi_l^0, H_n^0, H_m^0) J_{H_n^0 H_m^0}^{(1)(H_k^0 - \chi_l^0)} \\
&\quad - \frac{m_j}{4} \left( b_{H_m^0 H_n^0} - (R_{H_n^0 \chi_l^0} - 4 m_i^2 s) \right) \mathcal{A}(\chi_l^0, H_n^0, H_m^0) J_{H_n^0 H_m^0}^{(2)(H_k^0 - \chi_l^0)} \\
&\quad + \frac{1}{2} d m_{\chi_l^0} s L(\chi_l^0, H_n^0, H_m^0) J_{H_n^0 H_m^0}^{(3)(H_k^0 - \chi_l^0)} - \frac{m_i}{4} \left( b_{H_m^0 H_n^0} - (R_{H_n^0 \chi_l^0} \right. \\
&\quad \left. + 4 d s - \tau_{H_m^0 H_n^0} - \omega_{H_m^0 H_n^0}) \mathcal{A}(\chi_l^0, H_n^0, H_m^0) \right) J_{H_n^0 H_m^0}^{(4)(H_k^0 - \chi_l^0)} \quad (3.10)
\end{aligned}$$

$$\tilde{w}_{H_m^0, H_n^0}^{(H_k^0, \chi_l^0)} = \frac{1}{b_{H_m^0 H_n^0} (s - m_{H_k^0}^2)} \left( \tilde{w}_{H_m^0, H_n^0}^{st(H_k^0, \chi_l^0)} + \tilde{w}_{H_m^0, H_n^0}^{su(H_k^0, \chi_l^0)} \right) + c.c. \quad (3.11)$$

The functions  $\Gamma$ 's,  $J$ 's,  $I$ 's,  $f$ 's and  $g$ 's, used in the above expressions depend on mixing matrices and the couplings in the MSSM and are presented in the Appendix B. From the expression given above one can obtain specific final states by taking  $m, n=1, 2$ . Please note that in case of identical final states ( $H_1^0 H_1^0$  and  $H_2^0 H_2^0$ )  $\tilde{w}$  has to be multiplied by a symmetry factor  $\frac{1}{2}$ .

### 3.2 $\chi_i^0 \chi_j^0 \rightarrow A_1^0 A_1^0$

This process is the sum of s-channel diagrams though CP even Higgs boson ( $H_1^0$  and  $H_2^0$ ), the t and u-channel neutralino ( $\chi_k^0$ ,  $k = 1, \dots, 4$ ) exchange. The resulting cross section, including the s, t, u and possible interference processes among themselves, is given by (Similar to the  $\chi_i^0 \chi_j^0 \rightarrow H_m^0 H_n^0$ )

$$\tilde{w}_{A_1^0, A_1^0} = \sum_{k, l=1, 2} \tilde{w}_{A_1^0, A_1^0}^{(H_k^0, H_l^0)} + \sum_{k=1, 2} \sum_{l=1, 4} \tilde{w}_{A_1^0, A_1^0}^{(H_k^0, \chi_l^0)} + \sum_{k, l=1, 4} \tilde{w}_{A_1^0, A_1^0}^{(\chi_k^0, \chi_l^0)} \quad (3.12)$$



- CP-odd Higgs boson ( $A_1^0 - A_1^0$ ) exchange:

$$\tilde{w}_{A_1^0, A_1^0}^{(H_k^0, H_l^0)} = \frac{1}{2} \frac{C_{H_k^0 A_1^0 A_1^0} C_{H_l^0 A_1^0 A_1^0}^*}{(s - m_{H_k^0}^2)(s - m_{H_l^0}^2)} \left( f_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)}(s - m_i^2 - m_j^2) - 2 m_i m_j g_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} \right) \quad (3.13)$$

- Neutralino ( $\chi_k^0 - \chi_l^0$ ) exchange:

$$\begin{aligned} \tilde{w}_{A_1^0, A_1^0}^{t(\chi_k^0, \chi_l^0)} &= 4 d m_{\chi_k^0} m_{\chi_l^0} \Gamma_{A_1^0 A_1^0}^{1(\chi_k^0 - \chi_l^0)} + \beta_1(A_1^0, A_1^0, \chi_k^0, \chi_l^0) \Gamma_{A_1^0 A_1^0}^{2(\chi_k^0 - \chi_l^0)} \\ &\quad - 4 m_i m_j m_{\chi_k^0} m_{\chi_l^0} \Gamma_{A_1^0 A_1^0}^{3(\chi_k^0 - \chi_l^0)} - m_i m_j \beta_2(A_1^0, A_1^0, \chi_k^0, \chi_l^0) \Gamma_{A_1^0 A_1^0}^{4(\chi_k^0 - \chi_l^0)} \\ &\quad - m_j (m_{\chi_k^0} \Gamma_{A_1^0 A_1^0}^{5(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{A_1^0 A_1^0}^{6(\chi_k^0 - \chi_l^0)}) \beta_3(A_1^0, A_1^0, \chi_k^0, \chi_l^0) \\ &\quad + (4 d m_i - \frac{m_i \omega_{A_1^0 A_1^0}}{s}) (m_{\chi_k^0} \Gamma_{A_1^0 A_1^0}^{7(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{A_1^0 A_1^0}^{8(\chi_k^0 - \chi_l^0)}) \end{aligned} \quad (3.14)$$

$$\begin{aligned} \tilde{w}_{A_1^0, A_1^0}^{u(\chi_k^0, \chi_l^0)} &= 4 d m_{\chi_k^0} m_{\chi_l^0} \Gamma_{A_1^0 A_1^0}^{1(\chi_k^0 - \chi_l^0)} + \beta_1(A_1^0, A_1^0, \chi_k^0, \chi_l^0) \Gamma_{A_1^0 A_1^0}^{2(\chi_k^0 - \chi_l^0)} \\ &\quad - 4 m_i m_j m_{\chi_k^0} m_{\chi_l^0} \Gamma_{A_1^0 A_1^0}^{3(\chi_k^0 - \chi_l^0)} - m_i m_j \beta_2(A_1^0, A_1^0, \chi_k^0, \chi_l^0) \Gamma_{A_1^0 A_1^0}^{4(\chi_k^0 - \chi_l^0)} \\ &\quad - m_j (m_{\chi_k^0} \Gamma_{A_1^0 A_1^0}^{5(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{A_1^0 A_1^0}^{6(\chi_k^0 - \chi_l^0)}) \beta_3(A_1^0, A_1^0, \chi_k^0, \chi_l^0) \\ &\quad + (4 d m_i - \frac{m_i \omega_{A_1^0 A_1^0}}{s}) (m_{\chi_k^0} \Gamma_{A_1^0 A_1^0}^{7(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{A_1^0 A_1^0}^{8(\chi_k^0 - \chi_l^0)}) \end{aligned} \quad (3.15)$$

$$\begin{aligned} \tilde{w}_{A_1^0, A_1^0}^{t-u(\chi_k^0, \chi_l^0)} &= d m_{\chi_k^0} m_{\chi_l^0} s^2 J_1(A_1^0, A_1^0, \chi_k^0, \chi_l^0) I_{A_1^0 A_1^0}^{1(\chi_k^0 - \chi_l^0)} + J_2(A_1^0, A_1^0, \chi_k^0, \chi_l^0) \\ &\quad I_{A_1^0 A_1^0}^{2(\chi_k^0 - \chi_l^0)} - m_i m_j m_{\chi_k^0} m_{\chi_l^0} s^2 J_1(A_1^0, A_1^0, \chi_k^0, \chi_l^0) I_{A_1^0 A_1^0}^{3(\chi_k^0 - \chi_l^0)} \\ &\quad + m_i m_j s^2 (l_i/2 - d_{A_1^0 A_1^0} - m_i^2) J_1(A_1^0, A_1^0, \chi_k^0, \chi_l^0) I_{A_1^0 A_1^0}^{4(\chi_k^0 - \chi_l^0)} \\ &\quad + \frac{1}{4} m_j m_{\chi_k^0} s J_3(A_1^0, A_1^0, \chi_k^0, \chi_l^0) I_{A_1^0 A_1^0}^{5(\chi_k^0 - \chi_l^0)} + \frac{1}{4} m_j m_{\chi_l^0} s \\ &\quad J_3(A_1^0, A_1^0, \chi_l^0, \chi_k^0) I_{A_1^0 A_1^0}^{6(\chi_k^0 - \chi_l^0)} - \frac{1}{4} m_i m_{\chi_k^0} s J_4(A_1^0, A_1^0, \chi_k^0, \chi_l^0) \\ &\quad I_{A_1^0 A_1^0}^{7(\chi_k^0 - \chi_l^0)} + \frac{1}{4} m_i m_{\chi_l^0} s J_5(A_1^0, A_1^0, \chi_k^0, \chi_l^0) I_{A_1^0 A_1^0}^{8(\chi_k^0 - \chi_l^0)} \end{aligned} \quad (3.16)$$

$$\tilde{w}_{A_1^0, A_1^0}^{(\chi_k^0, \chi_l^0)} = \tilde{w}_{A_1^0, A_1^0}^{t(\chi_k^0, \chi_l^0)} + \tilde{w}_{A_1^0, A_1^0}^{u(\chi_k^0, \chi_l^0)} + \left( \tilde{w}_{A_1^0, A_1^0}^{t-u(\chi_k^0, \chi_l^0)} + c.c. \right) \quad (3.17)$$

- CP-odd Higgs boson ( $A_1^0$ ) and Neutralino ( $\chi_l^0$ ) Interference:

$$\begin{aligned} \tilde{w}_{A_1^0, A_1^0}^{st(H_k^0, \chi_l^0)} &= -\frac{1}{2} m_i m_j m_{\chi_l^0} s L(\chi_l^0, A_1^0, A_1^0) J_{A_1^0 A_1^0}^{(1)(H_k^0 - \chi_l^0)} \\ &\quad - \frac{m_j}{4} \left( b_{A_1^0 A_1^0} - (R_{H_m^0 \chi_l^0} - 4 m_i^2 s) \right) \mathcal{A}(\chi_l^0, A_1^0, A_1^0) J_{A_1^0 A_1^0}^{(2)(H_k^0 - \chi_l^0)} \\ &\quad + \frac{1}{2} d m_{\chi_l^0} s L(\chi_l^0, A_1^0, A_1^0) J_{A_1^0 A_1^0}^{(3)(H_k^0 - \chi_l^0)} - \frac{m_i}{4} \left( b_{A_1^0 A_1^0} - (R_{A_1^0 \chi_l^0} \right. \\ &\quad \left. + 4 d s - \tau_{A_1^0 A_1^0} - \omega_{A_1^0 A_1^0}) \mathcal{A}(\chi_l^0, A_1^0, A_1^0) \right) J_{A_1^0 A_1^0}^{(4)(H_k^0 - \chi_l^0)} \end{aligned} \quad (3.18)$$

$$\begin{aligned}
\tilde{w}_{A_1^0 A_1^0}^{su(H_k^0, \chi_l^0)} &= -\frac{1}{2} m_i m_j m_{\chi_l^0} s L(\chi_l^0, A_1^0, A_1^0) J_{A_1^0 A_1^0}^{(1)(H_k^0 - \chi_l^0)} \\
&\quad - \frac{m_j}{4} \left( b_{A_1^0 A_1^0} - (R_{A_1^0 \chi_l^0} - 4 m_i^2 s) \right) \mathcal{A}(\chi_l^0, A_1^0, A_1^0) J_{A_1^0 A_1^0}^{(2)(H_k^0 - \chi_l^0)} \\
&\quad + \frac{1}{2} d m_{\chi_l^0} s L(\chi_l^0, A_1^0, A_1^0) J_{A_1^0 A_1^0}^{(3)(H_k^0 - \chi_l^0)} - \frac{m_i}{4} \left( b_{A_1^0 A_1^0} - (R_{A_1^0 \chi_l^0} \right. \\
&\quad \left. + 4 d s - \tau_{A_1^0 A_1^0} - \omega_{A_1^0 A_1^0}) \mathcal{A}(\chi_l^0, A_1^0, A_1^0) \right) J_{A_1^0 A_1^0}^{(4)(H_k^0 - \chi_l^0)} \quad (3.19)
\end{aligned}$$

$$\tilde{w}_{A_1^0 A_1^0}^{(H_k^0, \chi_l^0)} = \frac{1}{b_{A_1^0 A_1^0}(s - m_{H_k^0}^2)} \left( \tilde{w}_{A_1^0 A_1^0}^{st(H_k^0, \chi_l^0)} + \tilde{w}_{A_1^0 A_1^0}^{su(H_k^0, \chi_l^0)} \right) + c.c. \quad (3.20)$$

Note: As there are identical final states total cross section has to be multiplied by a factor  $\frac{1}{2}$

### 3.3 $\chi_i^0 \chi_j^0 \rightarrow H_1^0 A_1^0$

This process is sum of s-channel diagrams through CP-odd Higgs boson  $A_1^0$  and Z boson; t and u channel diagrams with neutralino exchange. The resulting cross section, including the s, t, u and possible interference processes among themselves, is given by

$$\tilde{w}_{H_1^0 A_1^0} = \tilde{w}_{H_1^0 A_1^0}^{(A_1^0, A_1^0)} + \tilde{w}_{H_1^0 A_1^0}^{(A_1^0, A_1^0)} + \tilde{w}_{H_1^0 A_1^0}^{(A_1^0, Z)} + \sum_{k,l=1,4} \tilde{w}_{H_1^0 A_1^0}^{(\chi_k^0, \chi_l^0)} + \sum_{l=1,4} \tilde{w}_{H_1^0 A_1^0}^{(A_1^0, \chi_l^0)} + \sum_{l=1,4} \tilde{w}_{H_1^0 A_1^0}^{(Z, \chi_l^0)} \quad (3.21)$$

- CP-odd Higgs boson  $A_1^0$  and Z exchange:

$$\tilde{w}_{H_1^0 A_1^0}^{(A_1^0, A_1^0)} = \frac{1}{2} \frac{|C_{A_1^0 H_1^0 A_1^0}|^2}{(s - m_{A_1^0}^2)^2} \left( f_{\chi_i^0 \chi_j^0}^{(A_1^0 - A_1^0)} (s - m_i^2 - m_j^2) - 2 m_i m_j g_{\chi_i^0 \chi_j^0}^{(A_1^0 - A_1^0)} \right) \quad (3.22)$$

$$\begin{aligned}
\tilde{w}_{H_1^0 A_1^0}^{(Z, Z)} &= \frac{1}{2} \frac{|C_{Z H_1^0 A_1^0}|^2}{(s - M_Z^2)^2 M_Z^4} \left( f_{\chi_i^0 \chi_j^0}^{(Z - Z)} \mathcal{B}_1(Z, H_1^0, A_1^0) - g_{\chi_i^0 \chi_j^0}^{(Z - Z)} \mathcal{B}_2(Z, H_1^0, A_1^0) \right) \\
\tilde{w}_{H_1^0 A_1^0}^{(A_1^0, Z)} &= \frac{1}{2} \frac{C_{A_1^0 H_1^0 A_1^0} C_{Z H_1^0 A_1^0}^*}{(s - m_{A_1^0}^2)^2 M_Z^2 s} \left( f_{\chi_i^0 \chi_j^0}^{(A_1^0 - Z)} m_j \tau_{H_1^0 A_1^0} - g_{\chi_i^0 \chi_j^0}^{(Z - Z)} m_i \tau_{A_1^0 H_1^0} \right) \\
&\quad (m_{H_1^0}^2 - m_{A_1^0}^2) + c.c. \quad (3.23)
\end{aligned}$$

- Neutralino  $(\chi_k^0 - \chi_l^0)$  exchange:

$$\begin{aligned}
\tilde{w}_{H_1^0 A_1^0}^{t(\chi_k^0, \chi_l^0)} &= 4 d m_{\chi_k^0} m_{\chi_l^0} \Gamma_{H_1^0 A_1^0}^{1(\chi_k^0 - \chi_l^0)} + \beta_1(H_1^0, A_1^0, \chi_k^0, \chi_l^0) \Gamma_{H_1^0 A_1^0}^{2(\chi_k^0 - \chi_l^0)} \\
&\quad - 4 m_i m_j m_{\chi_k^0} m_{\chi_l^0} \Gamma_{H_1^0 A_1^0}^{3(\chi_k^0 - \chi_l^0)} - m_i m_j \beta_2(H_1^0, A_1^0, \chi_k^0, \chi_l^0) \Gamma_{H_1^0 A_1^0}^{4(\chi_k^0 - \chi_l^0)} \\
&\quad - m_j (m_{\chi_k^0} \Gamma_{H_1^0 A_1^0}^{5(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{H_1^0 A_1^0}^{6(\chi_k^0 - \chi_l^0)}) \beta_3(H_1^0, A_1^0, \chi_k^0, \chi_l^0) \\
&\quad + \left( 4 d m_i - \frac{m_i \omega_{H_1^0 A_1^0}}{s} \right) \left( m_{\chi_k^0} \Gamma_{H_1^0 A_1^0}^{7(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{H_1^0 A_1^0}^{8(\chi_k^0 - \chi_l^0)} \right) \quad (3.24)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{A_1^0, H_1^0}^{u(\chi_k^0, \chi_l^0)} &= 4 d m_{\chi_k^0} m_{\chi_l^0} \Gamma_{A_1^0 A_1^0}^{1(\chi_k^0 - \chi_l^0)} + \beta_1(A_1^0, H_1^0, \chi_k^0, \chi_l^0) \Gamma_{A_1^0 A_1^0}^{2(\chi_k^0 - \chi_l^0)} \\
&\quad - 4 m_i m_j m_{\chi_k^0} m_{\chi_l^0} \Gamma_{A_1^0 A_1^0}^{3(\chi_k^0 - \chi_l^0)} - m_i m_j \beta_2(A_1^0, H_1^0, \chi_k^0, \chi_l^0) \Gamma_{A_1^0 A_1^0}^{4(\chi_k^0 - \chi_l^0)} \\
&\quad - m_j (m_{\chi_k^0} \Gamma_{A_1^0 A_1^0}^{5(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{A_1^0 A_1^0}^{6(\chi_k^0 - \chi_l^0)}) \beta_3(A_1^0, H_1^0, \chi_k^0, \chi_l^0) \\
&\quad + (4 d m_i - \frac{m_i \omega_{A_1^0 A_1^0}}{s}) (m_{\chi_k^0} \Gamma_{A_1^0 A_1^0}^{7(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{A_1^0 A_1^0}^{8(\chi_k^0 - \chi_l^0)}) \quad (3.25)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{H_1^0, A_1^0}^{t-u(\chi_k^0, \chi_l^0)} &= d m_{\chi_k^0} m_{\chi_l^0} s^2 J_1(H_1^0, A_1^0, \chi_k^0, \chi_l^0) I_{H_1^0 A_1^0}^{1(\chi_k^0 - \chi_l^0)} + J_2(H_1^0, A_1^0, \chi_k^0, \chi_l^0) \\
&\quad I_{H_1^0 A_1^0}^{2(\chi_k^0 - \chi_l^0)} - m_i m_j m_{\chi_k^0} m_{\chi_l^0} s^2 J_1(H_1^0, A_1^0, \chi_k^0, \chi_l^0) I_{H_1^0 A_1^0}^{3(\chi_k^0 - \chi_l^0)} \\
&\quad + m_i m_j s^2 (l_i/2 - d_{H_1^0 A_1^0} - m_i^2) J_1(H_1^0, A_1^0, \chi_k^0, \chi_l^0) I_{H_1^0 A_1^0}^{4(\chi_k^0 - \chi_l^0)} \\
&\quad + \frac{1}{4} m_j m_{\chi_k^0} s J_3(H_1^0, A_1^0, \chi_k^0, \chi_l^0) I_{H_1^0 A_1^0}^{5(\chi_k^0 - \chi_l^0)} + \frac{1}{4} m_j m_{\chi_l^0} s \\
&\quad J_3(H_1^0, A_1^0, \chi_l^0, \chi_k^0) I_{H_1^0 A_1^0}^{6(\chi_k^0 - \chi_l^0)} - \frac{1}{4} m_i m_{\chi_k^0} s J_4(H_1^0, A_1^0, \chi_k^0, \chi_l^0) \\
&\quad I_{H_1^0 A_1^0}^{7(\chi_k^0 - \chi_l^0)} + \frac{1}{4} m_i m_{\chi_l^0} s J_5(H_1^0, A_1^0, \chi_k^0, \chi_l^0) I_{H_1^0 A_1^0}^{8(\chi_k^0 - \chi_l^0)} \quad (3.26)
\end{aligned}$$

$$\tilde{w}_{H_1^0, A_1^0}^{(\chi_k^0, \chi_l^0)} = \tilde{w}_{H_1^0, A_1^0}^{t(\chi_k^0, \chi_l^0)} + \tilde{w}_{H_1^0, A_1^0}^{u(\chi_k^0, \chi_l^0)} + \left( \tilde{w}_{H_1^0, A_1^0}^{t-u(\chi_k^0, \chi_l^0)} + c.c. \right) \quad (3.27)$$

- CP-odd Higgs boson  $A_1^0$ , Z boson and Neutralino ( $\chi_l^0$ ) Interference:

$$\begin{aligned}
\tilde{w}_{H_1^0, A_1^0}^{st(Z, \chi_k^0)} &= \frac{C_{ZH_1^0 A_1^0}}{s - M_Z^2} \left( \frac{m_j m_{\chi_k^0}}{2 b_{H_1^0 A_1^0} M_Z^2} \mathcal{B}_3(Z, \chi_k^0, H_1^0, A_1^0) J_{H_1^0 A_1^0}^{(5)(Z - \chi_k^0)} \right. \\
&\quad + \frac{m_j m_{\chi_k^0}}{4 b_{H_1^0 A_1^0} M_Z^2} \mathcal{B}_4(Z, \chi_k^0, H_1^0, A_1^0) J_{H_1^0 A_1^0}^{(6)(Z - \chi_k^0)} + \mathcal{B}_5(Z, \chi_k^0, H_1^0, A_1^0) \\
&\quad \left. J_{H_1^0 A_1^0}^{(7)(Z - \chi_k^0)} + \frac{m_i m_j}{2 b_{H_1^0 A_1^0} M_Z^2} \mathcal{B}_6(Z, \chi_k^0, H_1^0, A_1^0) J_{H_1^0 A_1^0}^{(8)(Z - \chi_k^0)} \right) \quad (3.28)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{H_1^0, A_1^0}^{su(Z, \chi_k^0)} &= \frac{C_{ZH_1^0 A_1^0}}{s - M_Z^2} \left( -\frac{m_j m_{\chi_k^0}}{2 b_{H_1^0 A_1^0} M_Z^2} \mathcal{B}_3(Z, \chi_k^0, A_1^0, H_1^0) J_{A_1^0 H_1^0}^{(5)(Z - \chi_k^0)} \right. \\
&\quad - \frac{m_j m_{\chi_k^0}}{4 b_{H_1^0 A_1^0} M_Z^2} \mathcal{B}_4(Z, \chi_k^0, A_1^0, H_1^0) J_{A_1^0 H_1^0}^{(6)(Z - \chi_k^0)} - \mathcal{B}_5(Z, \chi_k^0, A_1^0, H_1^0) \\
&\quad \left. J_{A_1^0 H_1^0}^{(7)(Z - \chi_k^0)} + \frac{m_i m_j}{2 b_{H_1^0 A_1^0} M_Z^2} \mathcal{B}_6(Z, \chi_k^0, A_1^0, H_1^0) J_{A_1^0 H_1^0}^{(8)(Z - \chi_k^0)} \right) \quad (3.29)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{H_1^0, A_1^0}^{st(A_1^0, \chi_l^0)} &= -\frac{1}{2} m_i m_j m_{\chi_l^0} s L(\chi_l^0, H_1^0, A_1^0) J_{H_1^0 A_1^0}^{(1)(H_k^0 - \chi_l^0)} \\
&\quad - \frac{m_j}{4} \left( b_{H_1^0 A_1^0} - (R_{H_1^0 \chi_l^0} - 4 m_i^2 s) \right) \mathcal{A}(\chi_l^0, H_1^0, A_1^0) J_{H_1^0 A_1^0}^{(2)(H_k^0 - \chi_l^0)} \\
&\quad + \frac{1}{2} d m_{\chi_l^0} s L(\chi_l^0, H_1^0, A_1^0) J_{H_1^0 A_1^0}^{(3)(H_k^0 - \chi_l^0)} - \frac{m_i}{4} \left( b_{H_1^0 A_1^0} - (R_{H_1^0 \chi_l^0} \right. \\
&\quad \left. + 4 d s - \tau_{H_1^0 A_1^0} - \omega_{H_1^0 A_1^0}) \mathcal{A}(\chi_l^0, H_1^0, A_1^0) \right) J_{H_1^0 A_1^0}^{(4)(H_k^0 - \chi_l^0)} \quad (3.30)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{H_1^0, A_1^0}^{su(H_k^0, \chi_l^0)} &= -\frac{1}{2} m_i m_j m_{\chi_l^0} s L(\chi_l^0, A_1^0, H_1^0) J_{A_1^0 H_1^0}^{(1)(H_k^0 - \chi_l^0)} \\
&\quad - \frac{m_j}{4} \left( b_{H_1^0 A_1^0} - (R_{A_1^0 \chi_l^0} - 4 m_i^2 s) \right) \mathcal{A}(\chi_l^0, A_1^0, H_1^0) J_{A_1^0 H_1^0}^{(2)(H_k^0 - \chi_l^0)} \\
&\quad + \frac{1}{2} d m_{\chi_l^0} s L(\chi_l^0, A_1^0, H_1^0) J_{H_1^0 A_1^0}^{(3)(H_k^0 - \chi_l^0)} - \frac{m_i}{4} \left( b_{H_1^0 A_1^0} - (R_{A_1^0 \chi_l^0} \right. \\
&\quad \left. + 4 d s - \tau_{H_1^0 A_1^0} - \omega_{H_1^0 A_1^0}) \mathcal{A}(\chi_l^0, A_1^0, H_1^0) \right) J_{A_1^0 H_1^0}^{(4)(H_k^0 - \chi_l^0)} \quad (3.31)
\end{aligned}$$

$$\tilde{w}_{H_1^0, A_1^0}^{(A_1^0, \chi_l^0)} = \frac{1}{b_{H_1^0 A_1^0} (s - m_{A_1^0}^2)} \left( \tilde{w}_{H_1^0, A_1^0}^{st(A_1^0, \chi_l^0)} + \tilde{w}_{H_1^0, A_1^0}^{su(A_1^0, \chi_l^0)} \right) + c.c. \quad (3.32)$$

$$\tilde{w}_{H_1^0, A_1^0}^{(Z, \chi_l^0)} = \frac{1}{b_{H_1^0 A_1^0} (s - M_Z^2)} \left( \tilde{w}_{H_1^0, A_1^0}^{st(Z, \chi_l^0)} + \tilde{w}_{H_1^0, A_1^0}^{su(Z, \chi_l^0)} \right) + c.c. \quad (3.33)$$

### 3.4 $\chi_i^0 \chi_j^0 \rightarrow H_2^0 A_1^0$

This process is the sum of CP even Higgs boson  $H_k^0$  in s-channel and neutralino exchange in t and u channel. The resulting cross section, including the s, t, u and possible interference processes among themselves, is given by:

$$\tilde{w}_{H_2^0, A_1^0} = \sum_{k,l=1,2} \tilde{w}_{H_2^0, A_1^0}^{(H_k^0, H_l^0)} + \sum_{k,l=1,4} \tilde{w}_{H_2^0, A_1^0}^{(\chi_k^0, \chi_l^0)} + \sum_{k=1,2} \sum_{l=1,4} \tilde{w}_{H_2^0, A_1^0}^{(H_k^0, \chi_l^0)} \quad (3.34)$$

- CP-even Higgs boson ( $H_k^0, H_l^0$ ) exchange:

$$\tilde{w}_{H_2^0, A_1^0}^{(H_k^0, H_l^0)} = \frac{1}{2} \frac{C_{H_k^0 H_2^0 A_1^0} C_{H_l^0 H_2^0 A_1^0}^*}{(s - m_{H_k^0}^2)(s - m_{H_l^0}^2)} \left( f_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} (s - m_i^2 - m_j^2) - 2 m_i m_j g_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} \right) \quad (3.35)$$

- Neutralino ( $\chi_k^0 - \chi_l^0$ ) exchange:

$$\begin{aligned}
\tilde{w}_{H_2^0, A_1^0}^{t(\chi_k^0, \chi_l^0)} &= 4 d m_{\chi_k^0} m_{\chi_l^0} \Gamma_{H_2^0 A_1^0}^{1(\chi_k^0 - \chi_l^0)} + \beta_1 (H_2^0, A_1^0, \chi_k^0, \chi_l^0) \Gamma_{H_2^0 A_1^0}^{2(\chi_k^0 - \chi_l^0)} \\
&\quad - 4 m_i m_j m_{\chi_k^0} m_{\chi_l^0} \Gamma_{H_2^0 A_1^0}^{3(\chi_k^0 - \chi_l^0)} - m_i m_j \beta_2 (H_2^0, A_1^0, \chi_k^0, \chi_l^0) \Gamma_{H_2^0 A_1^0}^{4(\chi_k^0 - \chi_l^0)} \\
&\quad - m_j (m_{\chi_k^0} \Gamma_{H_2^0 A_1^0}^{5(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{H_2^0 A_1^0}^{6(\chi_k^0 - \chi_l^0)}) \beta_3 (H_2^0, A_1^0, \chi_k^0, \chi_l^0) \\
&\quad + \left( 4 d m_i - \frac{m_i \omega_{H_2^0 A_1^0}}{s} \right) (m_{\chi_k^0} \Gamma_{H_2^0 A_1^0}^{7(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{H_2^0 A_1^0}^{8(\chi_k^0 - \chi_l^0)}) \quad (3.36)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{A_1^0, H_2^0}^{u(\chi_k^0, \chi_l^0)} &= 4 d m_{\chi_k^0} m_{\chi_l^0} \Gamma_{A_1^0 H_2^0}^{1(\chi_k^0 - \chi_l^0)} + \beta_1 (A_1^0, H_2^0, \chi_k^0, \chi_l^0) \Gamma_{A_1^0 H_2^0}^{2(\chi_k^0 - \chi_l^0)} \\
&\quad - 4 m_i m_j m_{\chi_k^0} m_{\chi_l^0} \Gamma_{A_1^0 H_2^0}^{3(\chi_k^0 - \chi_l^0)} - m_i m_j \beta_2 (A_1^0, H_2^0, \chi_k^0, \chi_l^0) \Gamma_{A_1^0 H_2^0}^{4(\chi_k^0 - \chi_l^0)} \\
&\quad - m_j (m_{\chi_k^0} \Gamma_{A_1^0 H_2^0}^{5(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{A_1^0 H_2^0}^{6(\chi_k^0 - \chi_l^0)}) \beta_3 (A_1^0, H_2^0, \chi_k^0, \chi_l^0) \\
&\quad + \left( 4 d m_i - \frac{m_i \omega_{A_1^0 H_2^0}}{s} \right) (m_{\chi_k^0} \Gamma_{A_1^0 H_2^0}^{7(\chi_k^0 - \chi_l^0)} + m_{\chi_l^0} \Gamma_{A_1^0 H_2^0}^{8(\chi_k^0 - \chi_l^0)}) \quad (3.37)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{H_2^0, A_1^0}^{t-u(\chi_k^0, \chi_l^0)} &= d m_{\chi_k^0} m_{\chi_l^0} s^2 J_1(H_2^0, A_1^0, \chi_k^0, \chi_l^0) I_{H_2^0 A_1^0}^{1(\chi_k^0 - \chi_l^0)} + J_2(H_2^0, A_1^0, \chi_k^0, \chi_l^0) \\
&\quad I_{H_2^0 A_1^0}^{2(\chi_k^0 - \chi_l^0)} - m_i m_j m_{\chi_k^0} m_{\chi_l^0} s^2 J_1(H_2^0, A_1^0, \chi_k^0, \chi_l^0) I_{H_2^0 A_1^0}^{3(\chi_k^0 - \chi_l^0)} \\
&\quad + m_i m_j s^2 (l_i/2 - d_{H_2^0 A_1^0} - m_i^2) J_1(H_2^0, A_1^0, \chi_k^0, \chi_l^0) I_{H_2^0 A_1^0}^{4(\chi_k^0 - \chi_l^0)} \\
&\quad + \frac{1}{4} m_j m_{\chi_k^0} s J_3(H_2^0, A_1^0, \chi_k^0, \chi_l^0) I_{H_2^0 A_1^0}^{5(\chi_k^0 - \chi_l^0)} + \frac{1}{4} m_j m_{\chi_l^0} s \\
&\quad J_3(H_2^0, A_1^0, \chi_l^0, \chi_k^0) I_{H_2^0 A_1^0}^{6(\chi_k^0 - \chi_l^0)} - \frac{1}{4} m_i m_{\chi_k^0} s J_4(H_2^0, A_1^0, \chi_k^0, \chi_l^0) \\
&\quad I_{H_2^0 A_1^0}^{7(\chi_k^0 - \chi_l^0)} + \frac{1}{4} m_i m_{\chi_l^0} s J_5(H_2^0, A_1^0, \chi_k^0, \chi_l^0) I_{H_2^0 A_1^0}^{8(\chi_k^0 - \chi_l^0)} \quad (3.38)
\end{aligned}$$

$$\tilde{w}_{H_2^0, A_1^0}^{(\chi_k^0, \chi_l^0)} = \tilde{w}_{H_2^0, A_1^0}^{t(\chi_k^0, \chi_l^0)} + \tilde{w}_{H_2^0, A_1^0}^{u(\chi_k^0, \chi_l^0)} + \left( \tilde{w}_{H_2^0, A_1^0}^{t-u(\chi_k^0, \chi_l^0)} + c.c. \right) \quad (3.39)$$

- CP-even Higgs boson ( $H_k^0$ ) and Neutralino ( $\chi_l^0$ ) Interference:

$$\begin{aligned}
\tilde{w}_{H_2^0, A_1^0}^{st(H_k^0, \chi_l^0)} &= -\frac{1}{2} m_i m_j m_{\chi_l^0} s L(\chi_l^0, H_2^0, A_1^0) J_{H_2^0 A_1^0}^{(1)(H_k^0 - \chi_l^0)} \\
&\quad - \frac{m_j}{4} \left( b_{H_2^0 A_1^0} - (R_{H_2^0 \chi_l^0} - 4 m_i^2 s) \right) \mathcal{A}(\chi_l^0, H_2^0, A_1^0) J_{H_2^0 A_1^0}^{(2)(H_k^0 - \chi_l^0)} \\
&\quad + \frac{1}{2} d m_{\chi_l^0} s L(\chi_l^0, H_2^0, A_1^0) J_{H_2^0 A_1^0}^{(3)(H_k^0 - \chi_l^0)} - \frac{m_i}{4} \left( b_{H_2^0 A_1^0} - (R_{H_2^0 \chi_l^0} \right. \\
&\quad \left. + 4 d s - \tau_{H_2^0 A_1^0} - \omega_{H_2^0 A_1^0}) \mathcal{A}(\chi_l^0, H_2^0, A_1^0) \right) J_{H_2^0 A_1^0}^{(4)(H_k^0 - \chi_l^0)} \quad (3.40)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{H_2^0, A_1^0}^{su(H_k^0, \chi_l^0)} &= -\frac{1}{2} m_i m_j m_{\chi_l^0} s L(\chi_l^0, A_1^0, H_2^0) J_{A_1^0 A_1^0}^{(1)(H_k^0 - \chi_l^0)} \\
&\quad - \frac{m_j}{4} \left( b_{H_2^0 A_1^0} - (R_{H_2^0 \chi_l^0} - 4 m_i^2 s) \right) \mathcal{A}(\chi_l^0, A_1^0, H_2^0) J_{A_1^0 A_1^0}^{(2)(H_k^0 - \chi_l^0)} \\
&\quad + \frac{1}{2} d m_{\chi_l^0} s L(\chi_l^0, A_1^0, H_2^0) J_{A_1^0 A_1^0}^{(3)(H_k^0 - \chi_l^0)} - \frac{m_i}{4} \left( b_{H_2^0 A_1^0} - (R_{H_2^0 \chi_l^0} \right. \\
&\quad \left. + 4 d s - \tau_{H_2^0 A_1^0} - \omega_{H_2^0 A_1^0}) \mathcal{A}(\chi_l^0, A_1^0, H_2^0) \right) J_{A_1^0 H_2^0}^{(4)(H_k^0 - \chi_l^0)} \quad (3.41)
\end{aligned}$$

$$\tilde{w}_{H_2^0, A_1^0}^{(H_k^0, \chi_l^0)} = \frac{1}{b_{H_2^0 A_1^0} (s - m_{H_k^0}^2)} \left( \tilde{w}_{H_2^0, A_1^0}^{st(H_k^0, \chi_l^0)} + \tilde{w}_{H_2^0, A_1^0}^{su(H_k^0, \chi_l^0)} \right) + c.c. \quad (3.42)$$

### 3.5 $\chi_i^0 \chi_j^0 \rightarrow H^+ H^-$

This process is the sum of the s-channel Z and CP even Higgs boson ( $H_1^0$ ,  $H_2^0$ ) exchange and t- and u-channel chargino ( $\chi_k^\pm$ ,  $k = 1, 2$ ) exchange. Resulting cross section including considering interference via s, t and u channel diagrams is given by:

$$\tilde{w}_{H_1^+, H_1^-} = \sum_{k,l=1,2} \tilde{w}_{H_1^+, H_1^-}^{(H_k^0, H_l^0)} + \tilde{w}_{H_1^+, H_1^-}^{(Z, Z)} + \sum_{k=1,2} \sum_{l=1,2} \tilde{w}_{H_1^+, H_1^-}^{(H_k^0, \chi_l^\pm)} + \sum_{k,l=1,2} \tilde{w}_{H_1^+, H_1^-}^{(\chi_k^\pm, \chi_l^\pm)} \quad (3.43)$$

- CP even Higgs boson ( $H_k^0, H_l^0$ ) and Z exchange:

$$\tilde{w}_{H_1^+, H_1^-}^{s(H_k^0, H_l^0)} = \frac{1}{2} \frac{C_{H_k^0 H_1^+ H_1^-} C_{H_l^0 H_1^+ H_1^-}^*}{(s - m_{H_k^0}^2) (s - m_{H_l^0}^2)} \left( f_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} (s - m_i^2 - m_j^2) - 2 m_i m_j g_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} \right) \quad (3.44)$$

$$\tilde{w}_{H_1^+, H_1^-}^{s(Z, Z)} = \frac{1}{2} \frac{|C_{ZH_1^+ H_1^-}|^2}{(s - M_Z^2)^2 M_Z^4} \left( f_{\chi_i^0 \chi_j^0}^{(Z-Z)} \mathcal{B}_1(Z, H_1^+ H_1^-) - g_{\chi_i^0 \chi_j^0}^{(Z-Z)} \mathcal{B}_2(Z, H_1^+ H_1^-) \right) \quad (3.45)$$

$$\tilde{w}_{H_1^+, H_1^-}^{s(H_k^0, Z)} = 0 \quad (3.46)$$

- Chargino ( $\chi_k^\pm, \chi_l^\pm$ ) exchange:

$$\begin{aligned} \tilde{w}_{H_1^+, H_1^-}^{t(\chi_k^\pm, \chi_l^\pm)} &= 4d m_{\chi_k^\pm} m_{\chi_l^\pm} \Gamma_{H_1^+ H_1^-}^{1(\chi_k^\pm - \chi_l^\pm)} + \beta_1(H_1^+, H_1^-, \chi_k^\pm, \chi_l^\pm) \Gamma_{H_1^+ H_1^-}^{2(\chi_k^\pm - \chi_l^\pm)} \\ &\quad - 4m_i m_j m_{\chi_k^\pm} m_{\chi_l^\pm} \Gamma_{H_1^+ H_1^-}^{3(\chi_k^\pm - \chi_l^\pm)} - m_i m_j \beta_2(H_1^+, H_1^-, \chi_k^\pm, \chi_l^\pm) \Gamma_{H_1^+ H_1^-}^{4(\chi_k^\pm - \chi_l^\pm)} \\ &\quad - m_j \left( m_{\chi_k^\pm} \Gamma_{H_1^+ H_1^-}^{5(\chi_k^\pm - \chi_l^\pm)} + m_{\chi_l^\pm} \Gamma_{H_1^+ H_1^-}^{6(\chi_k^\pm - \chi_l^\pm)} \right) \beta_3(H_1^+, H_1^-, \chi_k^\pm, \chi_l^\pm) \\ &\quad + \left( 4d m_i - \frac{m_i \omega_{H_1^+ H_1^-}}{s} \right) \left( m_{\chi_k^\pm} \Gamma_{H_1^+ H_1^-}^{7(\chi_k^\pm - \chi_l^\pm)} + m_{\chi_l^\pm} \Gamma_{H_1^+ H_1^-}^{8(\chi_k^\pm - \chi_l^\pm)} \right) \quad (3.47) \end{aligned}$$

$$\begin{aligned} \tilde{w}_{H_1^+, H_1^-}^{u(\chi_k^\pm, \chi_l^\pm)} &= 4d m_{\chi_k^\pm} m_{\chi_l^\pm} \Gamma_{H_1^- H_1^+}^{1(\chi_k^\pm - \chi_l^\pm)*} + \beta_1(H_1^-, H_1^+, \chi_k^\pm, \chi_l^\pm) \Gamma_{H_1^- H_1^+}^{2(\chi_k^\pm - \chi_l^\pm)*} \\ &\quad - 4m_i m_j m_{\chi_k^\pm} m_{\chi_l^\pm} \Gamma_{H_1^- H_1^+}^{3(\chi_k^\pm - \chi_l^\pm)*} - m_i m_j \beta_2(H_1^-, H_1^+, \chi_k^\pm, \chi_l^\pm) \Gamma_{H_1^- H_1^+}^{4(\chi_k^\pm - \chi_l^\pm)*} \\ &\quad - m_j \left( m_{\chi_k^\pm} \Gamma_{H_1^- H_1^+}^{5(\chi_k^\pm - \chi_l^\pm)*} + m_{\chi_l^\pm} \Gamma_{H_1^- H_1^+}^{6(\chi_k^\pm - \chi_l^\pm)*} \right) \beta_3(H_1^-, H_1^+, \chi_k^\pm, \chi_l^\pm) \\ &\quad + \left( 4d m_i - \frac{m_i \omega_{H_1^- H_1^+}}{s} \right) \left( m_{\chi_k^\pm} \Gamma_{H_1^- H_1^+}^{7(\chi_k^\pm - \chi_l^\pm)*} + m_{\chi_l^\pm} \Gamma_{H_1^- H_1^+}^{8(\chi_k^\pm - \chi_l^\pm)*} \right) \quad (3.48) \end{aligned}$$

$$\begin{aligned} \tilde{w}_{H_1^+, H_1^-}^{t-u(\chi_k^\pm, \chi_l^\pm)} &= d m_{\chi_k^\pm} m_{\chi_l^\pm} s^2 J_1(H_1^+, H_1^-, \chi_k^\pm, \chi_l^\pm) I_{H_1^+, H_1^-}^{1(\chi_k^\pm - \chi_l^\pm)} + J_2(H_1^+, H_1^-, \chi_k^\pm, \chi_l^\pm) \\ &\quad I_{H_m^0 H_n^0}^{2(\chi_k^\pm - \chi_l^\pm)} - m_i m_j m_{\chi_k^\pm} m_{\chi_l^\pm} s^2 J_1(H_1^+, H_1^-, \chi_k^\pm, \chi_l^\pm) I_{H_1^+, H_1^-}^{3(\chi_k^\pm - \chi_l^\pm)} \\ &\quad + m_i m_j s^2 (l_i/2 - d_{H_1^+, H_1^-} - m_i^2) J_1(H_1^+, H_1^-, \chi_k^\pm, \chi_l^\pm) I_{H_1^+, H_1^-}^{4(\chi_k^\pm - \chi_l^\pm)} \\ &\quad + \frac{1}{4} m_j m_{\chi_k^\pm} s J_3(H_1^+, H_1^-, \chi_k^\pm, \chi_l^\pm) I_{H_1^+, H_1^-}^{5(\chi_k^\pm - \chi_l^\pm)} + \frac{1}{4} m_j m_{\chi_l^\pm} s \\ &\quad J_3(H_1^+, H_1^-, \chi_l^\pm, \chi_k^\pm) I_{H_1^+, H_1^-}^{6(\chi_k^\pm - \chi_l^\pm)} - \frac{1}{4} m_i m_{\chi_k^\pm} s J_4(H_1^+, H_1^-, \chi_k^\pm, \chi_l^\pm) \\ &\quad I_{H_1^+, H_1^-}^{7(\chi_k^\pm - \chi_l^\pm)} + \frac{1}{4} m_i m_{\chi_l^\pm} s J_5(H_1^+, H_1^-, \chi_k^\pm, \chi_l^\pm) I_{H_1^+, H_1^-}^{8(\chi_k^\pm - \chi_l^\pm)} \quad (3.49) \end{aligned}$$

$$\tilde{w}_{H_1^+, H_1^-}^{(\chi_k^\pm, \chi_l^\pm)} = \tilde{w}_{H_1^+, H_1^-}^{t(\chi_k^\pm, \chi_l^\pm)} + \tilde{w}_{H_1^+, H_1^-}^{u(\chi_k^\pm, \chi_l^\pm)} + \left( \tilde{w}_{H_1^+, H_1^-}^{t-u(\chi_k^\pm, \chi_l^\pm)} + c.c. \right) \quad (3.50)$$

- CP-even Higgs boson ( $H_k^0$ ) and Neutralino ( $\chi_l^\pm$ ) Interference:

$$\begin{aligned} \tilde{w}_{H_1^+, H_1^-}^{st(H_k^0, \chi_l^\pm)} &= -\frac{1}{2} m_i m_j m_{\chi_l^\pm} s L(\chi_l^\pm, H_1^+, H_1^-) J_{H_1^+ H_1^-}^{(1)(H_k^0 - \chi_l^\pm)} \\ &\quad - \frac{m_j}{4} \left( b_{H_1^+ H_1^-} - (R_{H_1^+ \chi_l^\pm} - 4m_i^2 s) \right) \mathcal{A}(\chi_l^\pm, H_1^+, H_1^-) J_{H_1^+ H_1^-}^{(2)(H_k^0 - \chi_l^\pm)} \\ &\quad + \frac{1}{2} d m_{\chi_l^\pm} s L(\chi_l^\pm, H_1^+, H_1^-) J_{H_1^+ H_1^-}^{(3)(H_k^0 - \chi_l^\pm)} - \frac{m_i}{4} \left( b_{H_1^+ H_1^-} - (R_{H_1^+ \chi_l^\pm} \right. \\ &\quad \left. + 4d s - \tau_{H_1^+ H_1^-} - \omega_{H_1^+ H_1^-}) \mathcal{A}(\chi_l^\pm, H_1^+, H_1^-) \right) J_{H_1^+ H_1^-}^{(4)(H_k^0 - \chi_l^\pm)} \quad (3.51) \end{aligned}$$

$$\begin{aligned}
\tilde{w}_{H_1^+, H_1^-}^{su(H_k^0, \chi_l^\pm)} &= -\frac{1}{2} m_i m_j m_{\chi_l^\pm} s L(\chi_l^\pm, H_1^-, H_1^+) J_{H_1^- H_1^+}^{(1)(H_k^0 - \chi_l^\pm)} \\
&\quad - \frac{m_j}{4} \left( b_{H_1^+ H_1^-} - (R_{H_1^- \chi_l^\pm} - 4 m_i^2 s) \right) \mathcal{A}(\chi_l^\pm, H_1^-, H_1^+) J_{H_1^- H_1^+}^{(2)(H_k^0 - \chi_l^\pm)} \\
&\quad + \frac{1}{2} d m_{\chi_l^\pm} s L(\chi_l^\pm, H_1^-, H_1^+) J_{H_1^+ H_1^-}^{(3)(H_k^0 - \chi_l^\pm)} - \frac{m_i}{4} \left( b_{H_1^+ H_1^-} - (R_{H_1^- \chi_l^\pm} \right. \\
&\quad \left. + 4 d s - \tau_{H_1^+ H_1^-} - \omega_{H_1^+ H_1^-}) \mathcal{A}(\chi_l^\pm, H_1^-, H_1^+) \right) J_{A_1^0 H_1^0}^{(4)(H_k^0 - \chi_l^\pm)} \quad (3.52)
\end{aligned}$$

$$\tilde{w}_{H_1^+, H_1^-}^{(H_k^0, \chi_l^\pm)} = \frac{1}{b_{H_1^+ H_1^-} (s - m_{H_k^0}^2)} \left( \tilde{w}_{H_1^+, H_1^-}^{st(H_k^0, \chi_l^\pm)} + \tilde{w}_{H_1^+, H_1^-}^{su(H_k^0, \chi_l^\pm)} \right) + c.c. \quad (3.53)$$

### 3.6 $\chi_i^0 \chi_j^0 \rightarrow W^\pm H^\mp$

This process is the sum of the s-channel CP even ( $H_1^0, H_2^0$ ), CP odd ( $A_1^0$ ) Higgs boson exchange and the t- and u-channel chargino ( $\chi_k^\pm, k = 1, 2$ ) exchange

$$\tilde{w}_{W^+ H_1^-} = \tilde{w}_{W^+ H_1^-}^{(A_1^0, A_1^0)} + \sum_{k,l=1,2} \tilde{w}_{W^+ H_1^-}^{(H_k^0, H_l^0)} + \sum_{k=1,2} \tilde{w}_{W^+ H_1^-}^{(H_k^0, A_1^0)} + \sum_{k,l=1,2} \left( \tilde{w}_{W^+ H_1^-}^{(\chi_k^\pm, \chi_l^\pm)} + \tilde{w}_{W^+ H_1^-}^{(H_k^0, \chi_l^\pm)} \right) \quad (3.54)$$

- CP-even Higgs boson ( $H_k^0 - H_l^0$ ) and CP-odd Higgs boson ( $A_1^0$ ):

$$\tilde{w}_{W^+ H_1^-}^{(A_1^0, A_1^0)} = \frac{1}{2} \frac{|C_{A_1^0 W^+ H_1^-}|^2}{(s - m_{A_1^0}^2)^2} \left( f_{\chi_i^0 \chi_j^0}^{(A_1^0 - A_1^0)} d - 2 m_i m_j g_{\chi_i^0 \chi_j^0}^{(A_1^0 - A_1^0)} \right) (s - l_{W^+ H^-}) \quad (3.55)$$

$$\tilde{w}_{W^+ H_1^-}^{(H_k^0, H_l^0)} = \frac{1}{2} \frac{C_{H_k^0 W^+ H_1^-} C_{H_l^0 W^+ H_1^-}^*}{(s - m_{H_k^0}^2) (s - m_{H_l^0}^2)} \left( f_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} d - 2 m_i m_j g_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} \right) (s - l_{W^+ H^-}) \quad (3.56)$$

$$\tilde{w}_{W^+ H_1^-}^{(H_k^0, A_1^0)} = \frac{1}{2} \frac{C_{H_k^0 W^+ H_1^-} C_{A_1^0 W^+ H_1^-}^*}{(s - m_{H_k^0}^2) (s - m_{A_1^0}^2)} \left( f_{\chi_i^0 \chi_j^0}^{(H_k^0 - A_1^0)} d - 2 m_i m_j g_{\chi_i^0 \chi_j^0}^{(H_k^0 - A_1^0)} \right) (s - l_{W^+ H^-}) \quad (3.57)$$

- Chargino ( $\chi_k^\pm - \chi_l^\pm$ ) exchange:

$$\begin{aligned}
\tilde{w}_{W^+ H_1^-}^{t(\chi_k^\pm, \chi_l^\pm)} &= \Gamma_{W^+ H_1^-}^{1(\chi_k^\pm - \chi_l^\pm)} \mathcal{C}_1(\chi_k^\pm, \chi_l^\pm, W^+, H_1^-) + \Gamma_{W^+ H_1^-}^{2(\chi_k^\pm - \chi_l^\pm)} \mathcal{C}_2(\chi_k^\pm, \chi_l^\pm, W^+, H_1^-) \\
&\quad - 3 m_i m_j s^2 \left( M_w^2 \Gamma_{W^+ H_1^-}^{3(\chi_k^\pm - \chi_l^\pm)} + m_{\chi_k^\pm} m_{\chi_l^\pm} \Gamma_{W^+ H_1^-}^{4(\chi_k^\pm - \chi_l^\pm)} \right) \\
&\quad \mathcal{C}_3(\chi_k^\pm, \chi_l^\pm, W^+, H_1^-) - \left( \Gamma_{W^+ H_1^-}^{5(\chi_k^\pm - \chi_l^\pm)} m_{\chi_l^\pm} + \Gamma_{W^+ H_1^-}^{6(\chi_k^\pm - \chi_l^\pm)} m_{\chi_k^\pm} \right) \\
&\quad \mathcal{C}_4(\chi_k^\pm, \chi_l^\pm, W^+, H_1^-) + \left( \Gamma_{W^+ H_1^-}^{7(\chi_k^\pm - \chi_l^\pm)} m_{\chi_l^\pm} + \Gamma_{W^+ H_1^-}^{8(\chi_k^\pm - \chi_l^\pm)} m_{\chi_k^\pm} \right) \\
&\quad \mathcal{C}_5(\chi_k^\pm, \chi_l^\pm, W^+, H_1^-) + \left( \Gamma_{W^+ H_1^-}^{5(\chi_k^\pm - \chi_l^\pm)} m_{\chi_l^\pm} + \Gamma_{W^+ H_1^-}^{6(\chi_k^\pm - \chi_l^\pm)} m_{\chi_k^\pm} \right. \\
&\quad \left. - 2 \Gamma_{W^+ H_1^-}^{3(\chi_k^\pm - \chi_l^\pm)} m_i \right) \mathcal{C}_6(\chi_k^\pm, \chi_l^\pm, W^+, H_1^-) \quad (3.58)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{W^+H_1^-}^{u(\chi_k^\pm, \chi_l^\pm)} &= \left( \Gamma_{H_1^-W^+}^{1(\chi_k^\pm - \chi_l^\pm)} m_{\chi_k^\pm} + \Gamma_{H_1^-W^+}^{2(\chi_k^\pm - \chi_l^\pm)} m_{\chi_l^\pm} \right) \mathcal{C}_{11}(\chi_k^\pm, \chi_l^\pm, H_1^-, W^+) \\
&+ \left( \Gamma_{H_1^-W^+}^{3(\chi_k^\pm - \chi_l^\pm)} m_{\chi_k^\pm} + \Gamma_{H_1^-W^+}^{4(\chi_k^\pm - \chi_l^\pm)} m_{\chi_l^\pm} \right) \mathcal{C}_{10}(\chi_k^\pm, \chi_l^\pm, H_1^-, W^+) \\
&+ \left( \Gamma_{H_1^-W^+}^{3(\chi_k^\pm - \chi_l^\pm)} m_i m_{\chi_k^\pm} + \Gamma_{H_1^-W^+}^{4(\chi_k^\pm - \chi_l^\pm)} m_i m_{\chi_l^\pm} + \Gamma_{H_1^-W^+}^{5(\chi_k^\pm - \chi_l^\pm)} m_{\chi_l^\pm} m_{\chi_k^\pm} \right) \\
&\mathcal{C}_9(\chi_k^\pm, \chi_l^\pm, H_1^-, W^+) + \Gamma_{H_1^-W^+}^{6(\chi_k^\pm - \chi_l^\pm)} \mathcal{C}_8(\chi_k^\pm, \chi_l^\pm, H_1^-, W^+) + \Gamma_{H_1^-W^+}^{7(\chi_k^\pm - \chi_l^\pm)} \\
&\mathcal{C}_6(\chi_k^\pm, \chi_l^\pm, H_1^-, W^+) + \Gamma_{H_1^-W^+}^{8(\chi_k^\pm - \chi_l^\pm)} \mathcal{C}_7(\chi_k^\pm, \chi_l^\pm, H_1^-, W^+) \quad (3.59)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{W^+H_1^-}^{tu(\chi_k^\pm, \chi_l^\pm)} &= I_{W^+H_1^-}^{1(\chi_k^\pm - \chi_l^\pm)} J_6(W^+, H_1^-, \chi_k^\pm, \chi_l^\pm) + I_{W^+H_1^-}^{2(\chi_k^\pm - \chi_l^\pm)} J_7(W^+, H_1^-, \chi_k^\pm, \chi_l^\pm) \\
&+ I_{W^+H_1^-}^{3(\chi_k^\pm - \chi_l^\pm)} J_8(W^+, H_1^-, \chi_k^\pm, \chi_l^\pm) + I_{W^+H_1^-}^{4(\chi_k^\pm - \chi_l^\pm)} J_9(W^+, H_1^-, \chi_k^\pm, \chi_l^\pm) \\
&+ I_{W^+H_1^-}^{5(\chi_k^\pm - \chi_l^\pm)} J_{10}(W^+, H_1^-, \chi_k^\pm, \chi_l^\pm) + I_{W^+H_1^-}^{6(\chi_k^\pm - \chi_l^\pm)} J_{11}(W^+, H_1^-, \chi_k^\pm, \chi_l^\pm) \\
&+ I_{W^+H_1^-}^{7(\chi_k^\pm - \chi_l^\pm)} J_{12}(W^+, H_1^-, \chi_k^\pm, \chi_l^\pm) + I_{W^+H_1^-}^{8(\chi_k^\pm - \chi_l^\pm)} J_{13}(W^+, H_1^-, \chi_k^\pm, \chi_l^\pm) \quad (3.60)
\end{aligned}$$

$$\tilde{w}_{W^+H_1^-}^{(\chi_k^\pm, \chi_l^\pm)} = \tilde{w}_{W^+H_1^-}^{t(\chi_k^\pm, \chi_l^\pm)} + \tilde{w}_{W^+H_1^-}^{u(\chi_k^\pm, \chi_l^\pm)} + \left( \tilde{w}_{W^+H_1^-}^{tu(\chi_k^\pm, \chi_l^\pm)} + c.c. \right) \quad (3.61)$$

- Chargino ( $H_k^0 - \chi_l^\pm$ ) exchange:

$$\begin{aligned}
\tilde{w}_{W^+H_1^-}^{st(H_k^0, \chi_l^\pm)} &= \left( (-2d l_i + l_{ia} m_i^2) J_{W^+H_1^-}^{(5)(H_k^0 - \chi_l^\pm)} + J_{W^+H_1^-}^{(6)(H_k^0 - \chi_l^\pm)} l_i m_i m_j + J_{W^+H_1^-}^{(7)(H_k^0 - \chi_l^\pm)} \right. \\
&\quad \left. l_i m_j m_{\chi_l^\pm} - J_{W^+H_1^-}^{(8)(H_k^0 - \chi_l^\pm)} l_{ia} m_i m_{\chi_l^\pm} \right) \mathcal{C}_{12}(H_k^0, \chi_l^\pm, H_1^-, W^+) \\
&+ \left( \left( 2l_{H_1^-W^+} (l_i - l_{ia}) M_W^2 - 2d_{W^+H_1^-} l_i s \right) J_{W^+H_1^-}^{(5)(H_k^0 - \chi_l^\pm)} - 2d_{W^+H_1^-} s \right. \\
&\quad \left. \left( 2J_{W^+H_1^-}^{(6)(H_k^0 - \chi_l^\pm)} m_i m_j - 2J_{W^+H_1^-}^{(8)(H_k^0 - \chi_l^\pm)} m_i m_{\chi_l^\pm} + 2J_{W^+H_1^-}^{(7)(H_k^0 - \chi_l^\pm)} m_j m_{\chi_l^\pm} \right) \right) \\
&\quad \frac{-b_{W^+H_1^-} + (\tau_{W^+H_1^-} - R_{W^+\chi_k^\pm}) \mathcal{A}(\chi_l^\pm, W^+, H^-)}{16b_{W^+H_1^-} M_W^2 s (m_{H_k^0}^2 - s)} \quad (3.62)
\end{aligned}$$



$$\begin{aligned}
\tilde{w}_{W^+H_1^-}^{su(H_k^0, \chi_l^\pm)} = & \left( 4s \left( J_{H_1^-W^+}^{(6)(H_k^0-\chi_l^\pm)} \mathcal{C}_{13}(H_k^0, \chi_l^\pm, H_1^-, W^+) l_{ia} m_i m_{\chi_l^\pm} + J_{H_1^-W^+}^{(5)(H_k^0-\chi_l^\pm)} \right. \right. \\
& m_j s l_i m_{\chi_l^\pm} \left( l_{W^+H_1^-} - 6M_W^2 \right) + J_{H_1^-W^+}^{(8)(H_k^0-\chi_l^\pm)} m_i m_j s \left( (M_W^2 - m_{H_1^-}^2) l_i \right. \\
& + 6M_W^2 (M_W^2 - m_{H_1^-}^2 - m_i^2 + m_j^2) + 2b_{W^+H_1^-} s - l_{ia} s \left. \left. \right) + 2J_{H_1^-W^+}^{(5)(H_k^0-\chi_l^\pm)} \right. \\
& (2ds \mathcal{C}_{14}(H_k^0, \chi_l^\pm, H_1^-, W^+) + 2\mathcal{C}_{13}(H_k^0, \chi_l^\pm, H_1^-, W^+) l_{ia} m_i^2 s \\
& + l_{H_1^-W^+} l_i l_{ia} (M_W^2 - m_{H_1^-}^2) (M_W^2 - s) \left. \left. \right) \right) \frac{\mathcal{A}(\chi_l^\pm, H^-, W^+)}{16b_{W^+H_1^-} M_W^2 (m_{H_k^0}^2 - s) s} \\
& \left( \left( J_{H_1^-W^+}^{(8)(H_k^0-\chi_l^\pm)} m_i m_j - J_{H_1^-W^+}^{(6)(H_k^0-\chi_l^\pm)} m_i m_{\chi_l^\pm} + J_{H_1^-W^+}^{(5)(H_k^0-\chi_l^\pm)} m_j m_{\chi_l^\pm} \right) \right. \\
& 4s (2M_W^2 - s) - 2J_{H_1^-W^+}^{(7)(H_k^0-\chi_l^\pm)} \left( l_i (M_W^2 - m_{H_1^-}^2) (M_W^2 - s) + 4M_W^2 s \right. \\
& (2m_i^2 - m_j^2) - 2m_i^2 s^2 \left. \left. \right) \right) \\
& \frac{-b_{W^+H_1^-} + (\tau_{H_1^-W^+} - R_{H_1^- \chi_k^\pm}) \mathcal{A}(\chi_l^\pm, H^-, W^+)}{16b_{W^+H_1^-} M_W^2 (m_{H_k^0}^2 - s) s}
\end{aligned} \tag{3.63}$$

$$\tilde{w}_{W^+H_1^-}^{(H_k^0, \chi_l^\pm)} = \left( \tilde{w}_{W^+H_1^-}^{st(H_k^0, \chi_l^\pm)} + \tilde{w}_{W^+H_1^-}^{su(H_k^0, \chi_l^\pm)} \right) + c.c. \tag{3.64}$$

Note that the contributions to the final state  $W^- H^+$  and the final state  $W^+ H^-$  are same.

### 3.7 $\chi_i^0 \chi_j^0 \rightarrow ZH_n^0$

This process is sum of the s-channel CP odd Higgs boson and Z boson besides t and u channel neutralino exchange. Resulting cross section is obtained by taking the sum of the s, t, and u channel cross sections and possible cross sections obtained considering interference among themselves. And it is given by:

$$\tilde{w}_{Z, H_n^0} = \tilde{w}_{Z, H_n^0}^{s(A_1^0, A_1^0)} + \tilde{w}_{Z, H_n^0}^{s(Z, Z)} + \tilde{w}_{Z, H_n^0}^{s(A_1^0, Z)} + \sum_{k, l=1,4} \tilde{w}_{ZH_n^0}^{(\chi_k^0, \chi_l^0)} + \sum_{l=1,4} \tilde{w}_{Z, H_n^0}^{(Z, \chi_l^0)} + \sum_{l=1,4} \tilde{w}_{Z, H_n^0}^{(A_1^0, \chi_l^0)} \tag{3.65}$$

- CP-odd Higgs boson  $A_1^0$  and Z exchange:

$$\tilde{w}_{Z, H_n^0}^{s(A_1^0, A_1^0)} = \frac{1}{2} \frac{|C_{A_1^0 Z H_n^0}|^2}{(s - m_{A_1^0}^2)^2} \left( f_{\chi_i^0 \chi_j^0}^{(A_1^0 - A_1^0)} d - 2m_i m_j g_{\chi_i^0 \chi_j^0}^{(A_1^0 - A_1^0)} \right) (s - l_{ZH_n^0}) \tag{3.66}$$

$$\tilde{w}_{Z, H_n^0}^{s(Z, Z)} = \frac{|C_{ZZ H_n^0}|^2}{(s - M_Z^2)^2} \left( f_{\chi_i^0 \chi_j^0}^{(Z-Z)} \mathcal{B}_7(Z, Z, H_n^0) + m_i m_j g_{\chi_i^0 \chi_j^0}^{(Z-Z)} \mathcal{B}_8(Z, Z, H_n^0) \right) \tag{3.67}$$

$$\begin{aligned}
\tilde{w}_{Z, H_n^0}^{s(A_1^0, Z)} = & \frac{C_{ZZ H_n^0} C_{A_1^0 Z H_n^0}^*}{16M_Z^4 s (s - m_A^2)} \left( -l_i m_j f_{\chi_i^0 \chi_j^0}^{(Z-A_1^0)} + l_{ia} m_i g_{\chi_i^0 \chi_j^0}^{(Z-A_1^0)} \right) \\
& \left( 2d_{ZH_n^0} l_{ZH_n^0} + l_{ZH_n^0}^2 - 2M_Z^2 (l_{H_n^0 Z} + 2s) \right)
\end{aligned} \tag{3.68}$$

- Neutralino ( $\chi_k^0 - \chi_l^0$ ) exchange:

$$\begin{aligned}
\tilde{w}_{ZH_n^0}^{t(\chi_k^0, \chi_l^0)} &= \Gamma_{ZH_n^0}^{1(\chi_k^0 - \chi_l^0)} \mathcal{C}_1(\chi_k^0, \chi_l^0, Z, H_n^0) + \Gamma_{ZH_n^0}^{2(\chi_k^0 - \chi_l^0)} \mathcal{C}_2(\chi_k^0, \chi_l^0, Z, H_n^0) - 3 m_i m_j s^2 \\
&\quad \left( M_Z^2 \Gamma_{ZH_n^0}^{3(\chi_k^0 - \chi_l^0)} + m_{\chi_k^0} m_{\chi_l^0} \Gamma_{ZH_n^0}^{4(\chi_k^0 - \chi_l^0)} \right) \mathcal{C}_3(\chi_k^0, \chi_l^0, Z, H_n^0) - \left( \Gamma_{ZH_n^0}^{5(\chi_k^0 - \chi_l^0)} m_{\chi_l^0} \right. \\
&\quad \left. + \Gamma_{ZH_n^0}^{6(\chi_k^\pm - \chi_l^\pm)} m_{\chi_k^\pm} \right) \mathcal{C}_4(\chi_k^0, \chi_l^0, Z_n, H_n^0) + \left( \Gamma_{ZH_n^0}^{7(\chi_k^0 - \chi_l^0)} m_{\chi_l^0} + \Gamma_{ZH_n^0}^{8(\chi_k^0 - \chi_l^0)} m_{\chi_k^0} \right) \\
&\quad \mathcal{C}_5(\chi_k^0, \chi_l^0, Z_n, H_n^0) + \left( \Gamma_{ZH_n^0}^{5(\chi_k^0 - \chi_l^0)} m_{\chi_l^0} + \Gamma_{ZH_n^0}^{6(\chi_k^0 - \chi_l^0)} m_{\chi_k^0} - 2 \Gamma_{ZH_n^0}^{3(\chi_k^0 - \chi_l^0)} m_i \right) \\
&\quad \mathcal{C}_6(\chi_k^0, \chi_l^0, Z_n, H_n^0) \tag{3.69}
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{ZH_n^0}^{u(\chi_k^0, \chi_l^0)} &= \left( \Gamma_{H_n^0 Z}^{1(\chi_k^0 - \chi_l^0)} m_{\chi_k^0} + \Gamma_{H_n^0 Z}^{2(\chi_k^0 - \chi_l^0)} m_{\chi_l^0} \right) \mathcal{C}_{11}(\chi_k^0, \chi_l^0, H_n^0, Z) \\
&\quad + \left( \Gamma_{H_n^0 Z}^{3(\chi_k^0 - \chi_l^0)} m_{\chi_k^0} + \Gamma_{H_n^0 Z}^{4(\chi_k^0 - \chi_l^0)} m_{\chi_l^0} \right) \mathcal{C}_{10}(\chi_k^0, \chi_l^0, H_n^0, Z) + \left( \Gamma_{H_n^0 Z}^{3(\chi_k^0 - \chi_l^0)} \right. \\
&\quad \left. m_i m_{\chi_k^0} + \Gamma_{H_n^0 Z}^{4(\chi_k^0 - \chi_l^0)} m_i m_{\chi_l^0} + \Gamma_{H_n^0 Z}^{5(\chi_k^0 - \chi_l^0)} m_{\chi_l^0} m_{\chi_k^0} \right) \mathcal{C}_9(\chi_k^0, \chi_l^0, H_n^0, Z) \\
&\quad + \Gamma_{H_n^0 Z}^{6(\chi_k^0 - \chi_l^0)} \mathcal{C}_8(\chi_k^0, \chi_l^0, H_n^0, Z) + \Gamma_{H_n^0 Z}^{7(\chi_k^0 - \chi_l^0)} \mathcal{C}_6(\chi_k^0, \chi_l^0, H_n^0, Z) \\
&\quad + \Gamma_{H_n^0 Z}^{8(\chi_k^0 - \chi_l^0)} \mathcal{C}_7(\chi_k^0, \chi_l^0, H_n^0, Z) \tag{3.70}
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{ZH_n^0}^{tu(\chi_k^0, \chi_l^0)} &= I_{ZH_n^0}^{1(\chi_k^0 - \chi_l^0)} J_6(Z, H_n^0, \chi_k^0, \chi_l^0) + I_{ZH_n^0}^{2(\chi_k^0 - \chi_l^0)} J_7(Z, H_n^0, \chi_k^0, \chi_l^0) \\
&\quad + I_{ZH_n^0}^{3(\chi_k^0 - \chi_l^0)} J_8(Z, H_n^0, \chi_k^0, \chi_l^0) + I_{ZH_n^0}^{4(\chi_k^0 - \chi_l^0)} J_9(Z, H_n^0, \chi_k^0, \chi_l^0) \\
&\quad + I_{ZH_n^0}^{5(\chi_k^0 - \chi_l^0)} J_{10}(Z, H_n^0, \chi_k^0, \chi_l^0) + I_{ZH_n^0}^{6(\chi_k^0 - \chi_l^0)} J_{11}(Z, H_n^0, \chi_k^0, \chi_l^0) \\
&\quad + I_{ZH_n^0}^{7(\chi_k^0 - \chi_l^0)} J_{12}(Z, H_n^0, \chi_k^0, \chi_l^0) + I_{ZH_n^0}^{8(\chi_k^0 - \chi_l^0)} J_{13}(Z, H_n^0, \chi_k^0, \chi_l^0) \tag{3.71}
\end{aligned}$$

$$\tilde{w}_{ZH_n^0}^{(\chi_k^0, \chi_l^0)} = \tilde{w}_{ZH_n^0}^{t(\chi_k^0, \chi_l^0)} + \tilde{w}_{ZH_n^0}^{u(\chi_k^0, \chi_l^0)} + \left( \tilde{w}_{ZH_n^0}^{tu(\chi_k^0, \chi_l^0)} + c.c. \right) \tag{3.72}$$

- Z boson, CP odd Higgs boson ( $A_1^0$ ) and neutralino ( $\chi_l^0$ ) exchange:

$$\begin{aligned}
\tilde{w}_{Z, H_n^0}^{st(Z, \chi_l^0)} &= \frac{C_{ZZH_n^0}}{s - M_Z^2} \left( \mathcal{B}_9(Z, \chi_l^0, Z, H_n^0) J_{ZH_n^0}^{(5)(Z - \chi_l^0)} + \mathcal{B}_{10}(Z, \chi_l^0, Z, H_n^0) J_{ZH_n^0}^{(6)(Z - \chi_l^0)} \right. \\
&\quad \left. + \mathcal{B}_{11}(Z, \chi_l^0, Z, H_n^0) J_{ZH_n^0}^{(7)(Z - \chi_l^0)} + \mathcal{B}_{12}(Z, \chi_l^0, Z, H_n^0) J_{ZH_n^0}^{(8)(Z - \chi_l^0)} \right) \tag{3.73}
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{Z, H_n^0}^{su(Z, \chi_l^0)} &= \frac{C_{ZZH_n^0}}{s - M_Z^2} \left( \mathcal{B}_{13}(Z, \chi_l^0, H_n^0, Z) J_{H_n^0 Z}^{(5)(Z - \chi_l^0)} + \mathcal{B}_{14}(Z, \chi_l^0, H_n^0, Z) J_{H_n^0 Z}^{(6)(Z - \chi_l^0)} \right. \\
&\quad \left. + \mathcal{B}_{15}(Z, \chi_l^0, H_n^0, Z) J_{H_n^0 Z}^{(7)(Z - \chi_l^0)} + \mathcal{B}_{16}(Z, \chi_l^0, H_n^0, Z) J_{H_n^0 Z}^{(8)(Z - \chi_l^0)} \right) \tag{3.74}
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{ZH_n^0}^{st(A_1^0, \chi_l^0)} = & \left( (-2dl_i + l_{ia} m_i^2) J_{ZH_n^0}^{(5)(A_1^0 - \chi_l^0)} + J_{ZH_n^0}^{(6)(A_1^0 - \chi_l^0)} l_i m_i m_j + J_{ZH_n^0}^{(7)(A_1^0 - \chi_l^0)} \right. \\
& l_i m_j m_{\chi_l^0} - J_{ZH_n^0}^{(8)(A_1^0 - \chi_l^0)} l_{ia} m_i m_{\chi_l^0} \Big) \mathcal{C}_{12}(A_1^0, \chi_l^0, H_n^0, Z) \\
& + \left( (2l_{H_n^0 Z} (l_i - l_{ia}) M_Z^2 - 2d_{ZH_n^0} l_i s) J_{ZH_n^0}^{(5)(A_1^0 - \chi_l^0)} - 2d_{ZH_n^0} s \right. \\
& \left. \left( 2J_{ZH_n^0}^{(6)(A_1^0 - \chi_l^0)} m_i m_j - 2J_{ZH_n^0}^{(8)(A_1^0 - \chi_l^0)} m_i m_{\chi_l^0} + 2J_{ZH_n^0}^{(7)(A_1^0 - \chi_l^0)} m_j m_{\chi_l^0} \right) \right) \\
& \frac{-b_{ZH_n^0} + (\tau_{ZH_n^0} - R_{Z\chi_k^0}) \mathcal{A}(\chi_l^0, Z, H_n^0)}{16b_{ZH_n^0} M_Z^2 s (m_{A_1^0}^2 - s)} \quad (3.75)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{ZH_n^0}^{su(A_1^0, \chi_l^0)} = & \left( 4s \left( J_{H_n^0 Z}^{(6)(A_1^0 - \chi_l^0)} \mathcal{C}_{13}(A_1^0, \chi_l^0, H_n^0, Z) l_{ia} m_i m_{\chi_l^0} + J_{H_n^0 Z}^{(5)(A_1^0 - \chi_l^0)} \right. \right. \\
& m_j s l_i m_{\chi_l^0} (l_{ZH_n^0} - 6M_Z^2) + J_{H_n^0 Z}^{(8)(A_1^0 - \chi_l^0)} m_i m_j s \left. \left( (M_Z^2 - m_{H_n^0}^2) l_i \right. \right. \\
& \left. \left. + 6M_Z^2 (M_Z^2 - m_{H_n^0}^2 - m_i^2 + m_j^2) + 2b_{ZH_n^0} s - l_{ia} s \right) \right) + 2J_{H_n^0 Z}^{(5)(A_1^0 - \chi_l^0)} \\
& (2ds \mathcal{C}_{14}(A_1^0, \chi_l^0, H_n^0, Z) + 2\mathcal{C}_{13}(A_1^0, \chi_l^0, H_n^0, Z) l_{ia} m_i^2 s \\
& + l_{H_n^0 Z} l_i l_{ia} (M_Z^2 - m_{H_n^0}^2) (M_Z^2 - s) \Big) \frac{\mathcal{A}(\chi_l^0, H_n^0, Z)}{16b_{ZH_n^0} M_Z^2 (m_{A_1^0}^2 - s) s} \\
& \left( \left( J_{H_n^0 Z}^{(8)(A_1^0 - \chi_l^0)} m_i m_j - J_{H_n^0 Z}^{(6)(A_1^0 - \chi_l^0)} m_i m_{\chi_l^0} + J_{H_n^0 Z}^{(5)(A_1^0 - \chi_l^0)} m_j m_{\chi_l^0} \right) \right. \\
& 4s (2M_Z^2 - s) - 2J_{H_n^0 Z}^{(7)(A_1^0 - \chi_l^0)} \left( l_i (M_Z^2 - m_{H_n^0}^2) (M_Z^2 - s) + 4M_Z^2 s \right. \\
& \left. \left. (2m_i^2 - m_j^2) - 2m_i^2 s^2 \right) \right) \\
& \frac{-b_{ZH_n^0} + (\tau_{H_n^0 Z} - R_{H_n^0 \chi_k^0}) \mathcal{A}(\chi_l^0, H_n^0, Z)}{16b_{ZH_n^0} M_Z^2 (m_{A_1^0}^2 - s) s} \quad (3.76)
\end{aligned}$$

$$\tilde{w}_{ZH_n^0}^{(A_1^0, \chi_l^0)} = \left( \tilde{w}_{ZH_n^0}^{st(A_1^0, \chi_l^0)} + \tilde{w}_{ZH_n^0}^{su(A_1^0, \chi_l^0)} \right) + c.c. \quad (3.77)$$

$$\tilde{w}_{Z, H_n^0}^{(Z, \chi_l^0)} = \left( \tilde{w}_{Z, H_n^0}^{st(Z, \chi_l^0)} + \tilde{w}_{Z, H_n^0}^{su(Z, \chi_l^0)} \right) + c.c. \quad (3.78)$$

$$\tilde{w}_{Z, H_n^0}^{(A_1^0, \chi_l^0)} = \left( \tilde{w}_{Z, H_n^0}^{st(A_1^0, \chi_l^0)} + \tilde{w}_{Z, H_n^0}^{su(A_1^0, \chi_l^0)} \right) + c.c. \quad (3.79)$$

### 3.8 $\chi_i^0 \chi_j^0 \rightarrow ZH_n^0$

This process is sum of the s-channel CP even Higgs boson besides t and u channel neutralino exchange. Resulting cross section is obtained by taking the sum of the s, t, and u channel cross sections and possible cross sections obtained considering interference among themselves. And it is given by:

$$\tilde{w}_{Z, A_1^0} = \sum_{k, l=1,2} \tilde{w}_{Z, A_1^0}^{s(H_k^0, H_l^0)} + \sum_{k, l=1,4} \tilde{w}_{ZH_n^0}^{(\chi_k^0, \chi_l^0)} + \sum_{k=1,2} \sum_{l=1,4} \tilde{w}_{H_k^0, A_1^0}^{(H_k^0, \chi_l^0)} \quad (3.80)$$

- CP-even Higgs boson  $H_k^0 - H_l^0$  exchange:

$$\tilde{w}_{ZA_1^0}^{(H_k^0, H_l^0)} = \frac{1}{2} \frac{C_{H_k^0 ZA_1^0} C_{H_l^0 ZA_1^0}^*}{(s - m_{H_k^0}^2)(s - m_{H_l^0}^2)} \left( f_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} d - 2 m_i m_j g_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} \right) (s - l_{ZA_1^0}) \quad (3.81)$$

- Neutralino ( $\chi_k^0 - \chi_l^0$ ) exchange:

$$\begin{aligned} \tilde{w}_{ZA_1^0}^{t(\chi_k^0, \chi_l^0)} &= \Gamma_{ZA_1^0}^{1(\chi_k^0 - \chi_l^0)} \mathcal{C}_1(\chi_k^0, \chi_l^0, Z, A_1^0) + \Gamma_{ZA_1^0}^{2(\chi_k^0 - \chi_l^0)} \mathcal{C}_2(\chi_k^0, \chi_l^0, Z, A_1^0) - 3 m_i m_j s^2 \\ &\quad \left( M_Z^2 \Gamma_{ZA_1^0}^{3(\chi_k^0 - \chi_l^0)} + m_{\chi_k^0} m_{\chi_l^0} \Gamma_{ZA_1^0}^{4(\chi_k^0 - \chi_l^0)} \right) \mathcal{C}_3(\chi_k^0, \chi_l^0, Z, A_1^0) - \left( \Gamma_{ZA_1^0}^{5(\chi_k^0 - \chi_l^0)} m_{\chi_l^0} \right. \\ &\quad \left. + \Gamma_{ZA_1^0}^{6(\chi_k^\pm - \chi_l^\pm)} m_{\chi_k^\pm} \right) \mathcal{C}_4(\chi_k^0, \chi_l^0, Z_n, A_1^0) + \left( \Gamma_{ZA_1^0}^{7(\chi_k^0 - \chi_l^0)} m_{\chi_l^0} + \Gamma_{ZA_1^0}^{8(\chi_k^0 - \chi_l^0)} m_{\chi_k^0} \right) \\ &\quad \mathcal{C}_5(\chi_k^0, \chi_l^0, Z_n, A_1^0) + \left( \Gamma_{ZA_1^0}^{5(\chi_k^0 - \chi_l^0)} m_{\chi_l^0} + \Gamma_{ZA_1^0}^{6(\chi_k^0 - \chi_l^0)} m_{\chi_k^0} - 2 \Gamma_{ZA_1^0}^{3(\chi_k^0 - \chi_l^0)} m_i \right) \\ &\quad \mathcal{C}_6(\chi_k^0, \chi_l^0, Z_n, A_1^0) \end{aligned} \quad (3.82)$$

$$\begin{aligned} \tilde{w}_{ZA_1^0}^{u(\chi_k^0, \chi_l^0)} &= \left( \Gamma_{A_1^0 Z}^{1(\chi_k^0 - \chi_l^0)} m_{\chi_k^0} + \Gamma_{A_1^0 Z}^{2(\chi_k^0 - \chi_l^0)} m_{\chi_l^0} \right) \mathcal{C}_{11}(\chi_k^0, \chi_l^0, A_1^0, Z) \\ &\quad + \left( \Gamma_{A_1^0 Z}^{3(\chi_k^0 - \chi_l^0)} m_{\chi_k^0} + \Gamma_{A_1^0 Z}^{4(\chi_k^0 - \chi_l^0)} m_{\chi_l^0} \right) \mathcal{C}_{10}(\chi_k^0, \chi_l^0, A_1^0, Z) + \left( \Gamma_{A_1^0 Z}^{3(\chi_k^0 - \chi_l^0)} \right. \\ &\quad \left. m_i m_{\chi_k^0} + \Gamma_{A_1^0 Z}^{4(\chi_k^0 - \chi_l^0)} m_i m_{\chi_l^0} + \Gamma_{A_1^0 Z}^{5(\chi_k^0 - \chi_l^0)} m_{\chi_l^0} m_{\chi_k^0} \right) \mathcal{C}_9(\chi_k^0, \chi_l^0, A_1^0, Z) \\ &\quad + \Gamma_{A_1^0 Z}^{6(\chi_k^0 - \chi_l^0)} \mathcal{C}_8(\chi_k^0, \chi_l^0, A_1^0, Z) + \Gamma_{A_1^0 Z}^{7(\chi_k^0 - \chi_l^0)} \mathcal{C}_6(\chi_k^0, \chi_l^0, A_1^0, Z) \\ &\quad + \Gamma_{A_1^0 Z}^{8(\chi_k^0 - \chi_l^0)} \mathcal{C}_7(\chi_k^0, \chi_l^0, A_1^0, Z) \end{aligned} \quad (3.83)$$

$$\begin{aligned} \tilde{w}_{ZA_1^0}^{tu(\chi_k^0, \chi_l^0)} &= I_{ZA_1^0}^{1(\chi_k^0 - \chi_l^0)} J_6(Z, A_1^0, \chi_k^0, \chi_l^0) + I_{ZA_1^0}^{2(\chi_k^0 - \chi_l^0)} J_7(Z, A_1^0, \chi_k^0, \chi_l^0) \\ &\quad + I_{ZA_1^0}^{3(\chi_k^0 - \chi_l^0)} J_8(Z, A_1^0, \chi_k^0, \chi_l^0) + I_{ZA_1^0}^{4(\chi_k^0 - \chi_l^0)} J_9(Z, A_1^0, \chi_k^0, \chi_l^0) \\ &\quad + I_{ZA_1^0}^{5(\chi_k^0 - \chi_l^0)} J_{10}(Z, A_1^0, \chi_k^0, \chi_l^0) + I_{ZA_1^0}^{6(\chi_k^0 - \chi_l^0)} J_{11}(Z, A_1^0, \chi_k^0, \chi_l^0) \\ &\quad + I_{ZA_1^0}^{7(\chi_k^0 - \chi_l^0)} J_{12}(Z, A_1^0, \chi_k^0, \chi_l^0) + I_{ZA_1^0}^{8(\chi_k^0 - \chi_l^0)} J_{13}(Z, A_1^0, \chi_k^0, \chi_l^0) \end{aligned} \quad (3.84)$$

$$\tilde{w}_{ZA_1^0}^{(\chi_k^0, \chi_l^0)} = \tilde{w}_{ZA_1^0}^{t(\chi_k^0, \chi_l^0)} + \tilde{w}_{ZA_1^0}^{u(\chi_k^0, \chi_l^0)} + \left( \tilde{w}_{ZA_1^0}^{tu(\chi_k^0, \chi_l^0)} + c.c. \right) \quad (3.85)$$

- CP even Higgs boson ( $H_k^0$ ) and neutralino ( $\chi_l^0$ ) exchange:

$$\begin{aligned} \tilde{w}_{ZA_1^0}^{st(H_k^0, \chi_l^0)} &= \left( (-2 d l_i + l_{ia} m_i^2) J_{ZA_1^0}^{(5)(H_k^0 - \chi_l^0)} + J_{ZA_1^0}^{(6)(H_k^0 - \chi_l^0)} l_i m_i m_j + J_{ZA_1^0}^{(7)(H_k^0 - \chi_l^0)} \right. \\ &\quad \left. l_i m_j m_{\chi_l^0} - J_{ZA_1^0}^{(8)(H_k^0 - \chi_l^0)} l_{ia} m_i m_{\chi_l^0} \right) \mathcal{C}_{12}(H_k^0, \chi_l^0, A_1^0, Z) \\ &\quad + \left( \left( 2 l_{A_1^0 Z} (l_i - l_{ia}) M_Z^2 - 2 d_{ZA_1^0} l_i s \right) J_{ZA_1^0}^{(5)(H_k^0 - \chi_l^0)} - 2 d_{ZA_1^0} s \right. \\ &\quad \left. \left( 2 J_{ZA_1^0}^{(6)(H_k^0 - \chi_l^0)} m_i m_j - 2 J_{ZA_1^0}^{(8)(H_k^0 - \chi_l^0)} m_i m_{\chi_l^0} + 2 J_{ZA_1^0}^{(7)(H_k^0 - \chi_l^0)} m_j m_{\chi_l^0} \right) \right) \\ &\quad \frac{-b_{ZA_1^0} + (\tau_{ZA_1^0} - R_{Z\chi_k^0}) \mathcal{A}(\chi_l^0, Z, A_1^0)}{16 b_{ZA_1^0} M_Z^2 s (m_{H_k^0}^2 - s)} \end{aligned} \quad (3.86)$$

$$\begin{aligned}
\tilde{w}_{ZA_1^0}^{su(H_k^0, \chi_l^0)} = & \left( 4s \left( J_{A_1^0 Z}^{(6)(H_k^0 - \chi_l^0)} \mathcal{C}_{13}(H_k^0, \chi_l^0, A_1^0, Z) l_{ia} m_i m_{\chi_l^0} + J_{A_1^0 Z}^{(5)(H_k^0 - \chi_l^0)} \right. \right. \\
& m_j s l_i m_{\chi_l^0} \left( l_{ZA_1^0} - 6M_Z^2 \right) + J_{A_1^0 Z}^{(8)(H_k^0 - \chi_l^0)} m_i m_j s \left( (M_Z^2 - m_{A_1^0}^2) l_i \right. \\
& + 6M_Z^2 (M_Z^2 - m_{A_1^0}^2 - m_i^2 + m_j^2) + 2b_{ZA_1^0} s - l_{ia} s \left. \left. \right) \right) + 2J_{A_1^0 Z}^{(5)(H_k^0 - \chi_l^0)} \\
& (2ds \mathcal{C}_{14}(H_k^0, \chi_l^0, A_1^0, Z) + 2\mathcal{C}_{13}(H_k^0, \chi_l^0, A_1^0, Z) l_{ia} m_i^2 s \\
& + l_{A_1^0 Z} l_i l_{ia} (M_Z^2 - m_{A_1^0}^2) (M_Z^2 - s) \left. \right) \frac{\mathcal{A}(\chi_l^0, A_1^0, Z)}{16b_{ZA_1^0} M_Z^2 (m_{H_k^0}^2 - s) s} \\
& \left( \left( J_{A_1^0 Z}^{(8)(H_k^0 - \chi_l^0)} m_i m_j - J_{A_1^0 Z}^{(6)(H_k^0 - \chi_l^0)} m_i m_{\chi_l^0} + J_{A_1^0 Z}^{(5)(H_k^0 - \chi_l^0)} m_j m_{\chi_l^0} \right) \right. \\
& 4s (2M_Z^2 - s) - 2J_{A_1^0 Z}^{(7)(H_k^0 - \chi_l^0)} \left( l_i (M_Z^2 - m_{A_1^0}^2) (M_Z^2 - s) + 4M_Z^2 s \right. \\
& (2m_i^2 - m_j^2) - 2m_i^2 s^2 \left. \left. \right) \right. \\
& \left. \frac{-b_{ZA_1^0} + (\tau_{A_1^0 Z} - R_{A_1^0 \chi_k^0}) \mathcal{A}(\chi_l^0, A_1^0, Z)}{16b_{ZA_1^0} M_Z^2 (m_{H_k^0}^2 - s) s} \right) \quad (3.87)
\end{aligned}$$

$$\tilde{w}_{ZA_1^0}^{(H_k^0, \chi_l^0)} = \left( \tilde{w}_{ZA_1^0}^{st(H_k^0, \chi_l^0)} + \tilde{w}_{ZA_1^0}^{su(H_k^0, \chi_l^0)} \right) + c.c. \quad (3.88)$$

### 3.9 $\chi_i^0 \chi_j^0 \rightarrow W^\pm W^\mp$

This process is the sum of s-channel diagram through CP even ( $H_1^0, H_2^0$ ), Z boson and t- and u-channel chargino ( $\chi_k^\pm, k = 1, 2$ ) exchange.

$$\tilde{w}_{W^\pm W^\mp} = \sum_{k,l=1,2} \tilde{w}_{W^\pm W^\mp}^{(H_k^0, H_l^0)} + \tilde{w}_{W^\pm W^\mp}^{(Z, Z)} + \sum_{k,l=1,2} \tilde{w}_{W^\pm W^\mp}^{(\chi_k^\pm, \chi_l^\pm)} + \sum_{k=1,2} \sum_{l=1,4} \tilde{w}_{W^\pm W^\mp}^{(H_k^0, \chi_l^\pm)} + \sum_{l=1,4} \tilde{w}_{W^\pm W^\mp}^{(Z, \chi_l^\pm)} \quad (3.89)$$

- CP-even Higgs boson ( $H_k^0 - H_l^0$ ) and CP-odd Higgs boson (Z):

$$\begin{aligned}
\tilde{w}_{WW}^{(H_k^0 - H_l^0)} = & \frac{C_{H_k^0 WW} C_{H_l^0 WW}^*}{(S - m_{H_k^0}^2)(S - m_{H_l^0}^2)} \frac{1}{8M_W^4} (12M_W^4 - 4M_W^2 S + S^2) \\
& (f_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} d - g_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} m_i m_j) \quad (3.90)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{WW}^{(Z-Z)} = & \frac{C_{ZWW} C_{ZWW}^*}{(S - M_Z^2)^2} \left( -\frac{f_{\chi_i^0 \chi_j^0}^{(Z-Z)}}{48M_W^2 S^2} (24l_i l_{ia} M_W^2 (4M_W^2 - S) S^2 \right. \\
& + b_{WW}^2 (12M_W^4 - 4M_W^2 S + S^2)) + \frac{g_{\chi_i^0 \chi_j^0}^{(Z-Z)} m_i m_j}{M_W^2} S (S - 4M_W^2) \\
& \left. + \frac{f_{\chi_i^0 \chi_j^0}^{(Z-Z)} d + g_{\chi_i^0 \chi_j^0}^{(Z-Z)} m_i m_j}{8M_W^4} (80M_W^6 - 36M_W^4 S - 8M_W^2 S^2 + 3S^3) \right) \quad (3.91)
\end{aligned}$$

$$\tilde{w}_{WW}^{(Z-H_l^0)} = 0 \quad (3.92)$$

- Chargino exchange:

$$\begin{aligned}
\tilde{w}_{WW}^{t(\chi_k^\pm - \chi_l^\pm)} &= \Gamma_{WW}^{1(\chi_k^\pm - \chi_l^\pm)} \mathcal{C}_{12}(\chi_k^\pm, \chi_l^\pm, W, W) + \Gamma_{WW}^{2(\chi_k^\pm - \chi_l^\pm)} \mathcal{C}_{13}(\chi_k^\pm, \chi_l^\pm, W, W) \\
&+ \Gamma_{WW}^{3(\chi_k^\pm - \chi_l^\pm)} \mathcal{C}_{14}(\chi_k^\pm, \chi_l^\pm, W, W) + \Gamma_{WW}^{4(\chi_k^\pm - \chi_l^\pm)} \mathcal{C}_{15}(\chi_k^\pm, \chi_l^\pm, W, W) \\
&+ \left( m_{\chi_l^\pm} \Gamma_{WW}^{5(\chi_k^\pm - \chi_l^\pm)} + m_{\chi_k^\pm} \Gamma_{WW}^{6(\chi_k^\pm - \chi_l^\pm)} \right) \mathcal{C}_{16}(\chi_k^\pm, \chi_l^\pm, W, W) \\
&+ \left( m_{\chi_l^\pm} \Gamma_{WW}^{7(\chi_k^\pm - \chi_l^\pm)} + m_{\chi_k^\pm} \Gamma_{WW}^{8(\chi_k^\pm - \chi_l^\pm)} \right) \mathcal{C}_{17}(\chi_k^\pm, \chi_l^\pm, W, W) \quad (3.93)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{WW}^{u(\chi_k^\pm - \chi_l^\pm)} &= \Gamma_{WW}^{1(\chi_k^\pm - \chi_l^\pm)} \mathcal{C}_{12}(\chi_k^\pm, \chi_l^\pm, W, W) + \Gamma_{WW}^{2(\chi_k^\pm - \chi_l^\pm)} \mathcal{C}_{13}(\chi_k^\pm, \chi_l^\pm, W, W) \\
&+ \Gamma_{WW}^{3(\chi_k^\pm - \chi_l^\pm)} \mathcal{C}_{14}(\chi_k^\pm, \chi_l^\pm, W, W) + \Gamma_{WW}^{4(\chi_k^\pm - \chi_l^\pm)} \mathcal{C}_{15}(\chi_k^\pm, \chi_l^\pm, W, W) \\
&+ \left( m_{\chi_l^\pm} \Gamma_{WW}^{5(\chi_k^\pm - \chi_l^\pm)} + m_{\chi_k^\pm} \Gamma_{WW}^{6(\chi_k^\pm - \chi_l^\pm)} \right) \mathcal{C}_{16}(\chi_k^\pm, \chi_l^\pm, W, W) \\
&+ \left( m_{\chi_l^\pm} \Gamma_{WW}^{7(\chi_k^\pm - \chi_l^\pm)} + m_{\chi_k^\pm} \Gamma_{WW}^{8(\chi_k^\pm - \chi_l^\pm)} \right) \mathcal{C}_{17}(\chi_k^\pm, \chi_l^\pm, W, W) \quad (3.94)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{WW}^{tu(\chi_k^\pm - \chi_l^\pm)} &= I_{WW}^{1(\chi_k^\pm - \chi_l^\pm)} J_{14}(\chi_k^\pm, \chi_l^\pm, W, W) + I_{WW}^{2(\chi_k^\pm - \chi_l^\pm)} J_{15}(\chi_k^\pm, \chi_l^\pm, W, W) \\
&+ I_{WW}^{3(\chi_k^\pm - \chi_l^\pm)} J_{16}(\chi_k^\pm, \chi_l^\pm, W, W) + I_{WW}^{4(\chi_k^\pm - \chi_l^\pm)} J_{17}(\chi_k^\pm, \chi_l^\pm, W, W) \\
&+ I_{WW}^{5(\chi_k^\pm - \chi_l^\pm)} J_{18}(\chi_k^\pm, \chi_l^\pm, W, W) + I_{WW}^{6(\chi_k^\pm - \chi_l^\pm)} J_{19}(\chi_k^\pm, \chi_l^\pm, W, W) \\
&+ I_{WW}^{7(\chi_k^\pm - \chi_l^\pm)} J_{20}(\chi_k^\pm, \chi_l^\pm, W, W) + I_{WW}^{8(\chi_k^\pm - \chi_l^\pm)} J_{21}(\chi_k^\pm, \chi_l^\pm, W, W) \quad (3.95)
\end{aligned}$$

- CP-even Higgs boson ( $H_k^0$ ), Z boson interference with chargino ( $\chi_l^\pm$ ) :

$$\begin{aligned}
\tilde{w}_{WW}^{st(H_k^0 - \chi_l^\pm)} &= \frac{C_{H_k^0 WW}}{s - m_{H_k^0}^2} \left( J_{WW}^{(5)(H_k^0 - \chi_l^\pm)} \mathcal{B}_{20}(H_k^0, \chi_l^\pm, W, W) + J_{WW}^{(6)(H_k^0 - \chi_l^\pm)} \right. \\
&\mathcal{B}_{21}(H_k^0, \chi_l^\pm, W, W) + J_{WW}^{(7)(H_k^0 - \chi_l^\pm)} \mathcal{B}_{22}(H_k^0, \chi_l^\pm, W, W) + J_{WW}^{(8)(H_k^0 - \chi_l^\pm)} \\
&\left. \mathcal{B}_{23}(H_k^0, \chi_l^\pm, W, W) \right) \quad (3.96)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{WW}^{su(H_k^0 - \chi_l^\pm)} &= \frac{C_{H_k^0 WW}}{s - m_{H_k^0}^2} \left( J_{WW}^{(5)(H_k^0 - \chi_l^\pm)*} \mathcal{B}_{20}(H_k^0, \chi_l^\pm, W, W) + J_{WW}^{(6)(H_k^0 - \chi_l^\pm)*} \right. \\
&\mathcal{B}_{21}(H_k^0, \chi_l^\pm, W, W) + J_{WW}^{(7)(H_k^0 - \chi_l^\pm)*} \mathcal{B}_{22}(H_k^0, \chi_l^\pm, W, W) + J_{WW}^{(8)(H_k^0 - \chi_l^\pm)*} \\
&\left. \mathcal{B}_{23}(H_k^0, \chi_l^\pm, W, W) \right) \quad (3.97)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{WW}^{st(Z - \chi_l^\pm)} &= \frac{C_{Z WW}}{s - M_Z^2} \left( J_{WW}^{(5)(Z - \chi_l^\pm)} \mathcal{B}_{24}(Z, \chi_l^\pm, W, W) + J_{WW}^{(6)(Z - \chi_l^\pm)} \right. \\
&\mathcal{B}_{25}(Z, \chi_l^\pm, W, W) + J_{WW}^{(7)(Z - \chi_l^\pm)} \mathcal{B}_{26}(Z, \chi_l^\pm, W, W) + J_{WW}^{(8)(Z - \chi_l^\pm)} \\
&\left. \mathcal{B}_{27}(Z, \chi_l^\pm, W, W) \right) \quad (3.98)
\end{aligned}$$

$$\begin{aligned}\tilde{w}_{WW}^{su(Z-\chi_l^\pm)} = & -\frac{C_{ZWW}}{s-M_Z^2} \left( J_{WW}^{(5)(Z-\chi_l^\pm)*} \mathcal{B}_{24}(Z, \chi_l^\pm, W, W) + J_{WW}^{(6)(Z-\chi_l^\pm)*} \right. \\ & \mathcal{B}_{25}(Z, \chi_l^\pm, W, W) + J_{WW}^{(7)(Z-\chi_l^\pm)*} \mathcal{B}_{26}(Z, \chi_l^\pm, W, W) + J_{WW}^{(8)(Z-\chi_l^\pm)*} \\ & \left. \mathcal{B}_{27}(Z, \chi_l^\pm, W, W) \right) \quad (3.99)\end{aligned}$$

$$\tilde{w}_{WW}^{(H_k^0, \chi_l^\pm)} = \left( \tilde{w}_{WW}^{st(H_k^0, \chi_l^\pm)} + \tilde{w}_{WW}^{su(H_k^0, \chi_l^\pm)} \right) + c.c. \quad (3.100)$$

$$\tilde{w}_{WW}^{(Z, \chi_l^\pm)} = \left( \tilde{w}_{WW}^{st(Z, \chi_l^\pm)} + \tilde{w}_{WW}^{su(Z, \chi_l^\pm)} \right) + c.c. \quad (3.101)$$

### 3.10 $\chi_i^0 \chi_j^0 \rightarrow ZZ$

This process is the sum of s-channel CP even ( $H_1^0, H_2^0$ ) and the t- and u-channel neutralino ( $\chi_k^0, k = 1, 4$ ) exchange. Resulting cross section is obtained by taking the sum of the s, t, and u channel cross sections and possible cross sections obtained considering interference among themselves. And it is given by:

$$\tilde{w}_{ZZ} = \sum_{k,l=1,2} \tilde{w}_{ZZ}^{(H_k^0, H_l^0)} + \sum_{k,l=1,2} \tilde{w}_{ZZ}^{(\chi_k^0, \chi_l^0)} + \sum_{l=1,4} \tilde{w}_{ZZ}^{(H_k^0, \chi_l^0)} \quad (3.102)$$

- CP-even Higgs boson ( $H_k^0 - H_l^0$ ) :

$$\begin{aligned}\tilde{w}_{ZZ}^{(H_k^0 - H_l^0)} = & \frac{C_{H_k^0 ZZ} C_{H_l^0 ZZ}^*}{(S - m_{H_k^0}^2)(S - m_{H_l^0}^2)} \frac{1}{8 M_Z^4} (12 M_Z^4 - 4 M_Z^2 S + S^2) \\ & (f_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} d - g_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} m_i m_j) \quad (3.103)\end{aligned}$$

- Chargino exchange:

$$\begin{aligned}\tilde{w}_{ZZ}^{t(\chi_k^0 - \chi_l^0)} = & \Gamma_{ZZ}^{1(\chi_k^0 - \chi_l^0)} \mathcal{C}_{12}(\chi_k^0, \chi_l^0, Z, Z) + \Gamma_{ZZ}^{2(\chi_k^0 - \chi_l^0)} \mathcal{C}_{13}(\chi_k^0, \chi_l^0, Z, Z) \\ & + \Gamma_{ZZ}^{3(\chi_k^0 - \chi_l^0)} \mathcal{C}_{14}(\chi_k^0, \chi_l^0, Z, Z) + \Gamma_{ZZ}^{4(\chi_k^0 - \chi_l^0)} \mathcal{C}_{15}(\chi_k^0, \chi_l^0, Z, Z) \\ & + \left( m_{\chi_l^0} \Gamma_{ZZ}^{5(\chi_k^0 - \chi_l^0)} + m_{\chi_k^0} \Gamma_{ZZ}^{6(\chi_k^0 - \chi_l^0)} \right) \mathcal{C}_{16}(\chi_k^0, \chi_l^0, Z, Z) \\ & + \left( m_{\chi_l^0} \Gamma_{ZZ}^{7(\chi_k^0 - \chi_l^0)} + m_{\chi_k^0} \Gamma_{ZZ}^{8(\chi_k^0 - \chi_l^0)} \right) \mathcal{C}_{17}(\chi_k^0, \chi_l^0, Z, Z) \quad (3.104)\end{aligned}$$

$$\begin{aligned}\tilde{w}_{ZZ}^{u(\chi_k^0 - \chi_l^0)} = & \Gamma_{ZZ}^{1(\chi_k^0 - \chi_l^0)} \mathcal{C}_{12}(\chi_k^0, \chi_l^0, Z, Z) + \Gamma_{ZZ}^{2(\chi_k^0 - \chi_l^0)} \mathcal{C}_{13}(\chi_k^0, \chi_l^0, Z, Z) \\ & + \Gamma_{ZZ}^{3(\chi_k^0 - \chi_l^0)} \mathcal{C}_{14}(\chi_k^0, \chi_l^0, Z, Z) + \Gamma_{ZZ}^{4(\chi_k^0 - \chi_l^0)} \mathcal{C}_{15}(\chi_k^0, \chi_l^0, Z, Z) \\ & + \left( m_{\chi_l^0} \Gamma_{ZZ}^{5(\chi_k^0 - \chi_l^0)} + m_{\chi_k^0} \Gamma_{ZZ}^{6(\chi_k^0 - \chi_l^0)} \right) \mathcal{C}_{16}(\chi_k^0, \chi_l^0, Z, Z) \\ & + \left( m_{\chi_l^0} \Gamma_{ZZ}^{7(\chi_k^0 - \chi_l^0)} + m_{\chi_k^0} \Gamma_{ZZ}^{8(\chi_k^0 - \chi_l^0)} \right) \mathcal{C}_{17}(\chi_k^0, \chi_l^0, Z, Z) \quad (3.105)\end{aligned}$$

$$\begin{aligned}\tilde{w}_{ZZ}^{tu(\chi_k^0 - \chi_l^0)} = & I_{ZZ}^{1(\chi_k^0 - \chi_l^0)} J_{14}(\chi_k^0, \chi_l^0, Z, Z) + I_{ZZ}^{2(\chi_k^0 - \chi_l^0)} J_{15}(\chi_k^0, \chi_l^0, Z, Z) \\ & + I_{ZZ}^{3(\chi_k^0 - \chi_l^0)} J_{16}(\chi_k^0, \chi_l^0, Z, Z) + I_{ZZ}^{4(\chi_k^0 - \chi_l^0)} J_{17}(\chi_k^0, \chi_l^0, Z, Z) \\ & + I_{ZZ}^{5(\chi_k^0 - \chi_l^0)} J_{18}(\chi_k^0, \chi_l^0, Z, Z) + I_{ZZ}^{6(\chi_k^0 - \chi_l^0)} J_{19}(\chi_k^0, \chi_l^0, Z, Z) \\ & + I_{ZZ}^{7(\chi_k^0 - \chi_l^0)} J_{20}(\chi_k^0, \chi_l^0, Z, Z) + I_{ZZ}^{8(\chi_k^0 - \chi_l^0)} J_{21}(\chi_k^0, \chi_l^0, Z, Z) \quad (3.106)\end{aligned}$$

- CP-even Higgs boson ( $H_k^0$ ) interference with neutralino ( $\chi_l^0$ ) :

$$\begin{aligned}\tilde{w}_{ZZ}^{st(H_k^0-\chi_l^0)} &= \frac{C_{H_k^0 ZZ}}{s - m_{H_k^0}^2} \left( J_{ZZ}^{(5)(H_k^0-\chi_l^0)} \mathcal{B}_{20}(H_k^0, \chi_l^0, Z, Z) + J_{ZZ}^{(6)(H_k^0-\chi_l^0)} \right. \\ &\quad \mathcal{B}_{21}(H_k^0, \chi_l^0, Z, Z) + J_{ZZ}^{(7)(H_k^0-\chi_l^0)} \mathcal{B}_{22}(H_k^0, \chi_l^0, Z, Z) + J_{ZZ}^{(8)(H_k^0-\chi_l^0)} \\ &\quad \left. \mathcal{B}_{23}(H_k^0, \chi_l^0, Z, Z) \right) \quad (3.107)\end{aligned}$$

$$\begin{aligned}\tilde{w}_{ZZ}^{su(H_k^0-\chi_l^0)} &= \frac{C_{H_k^0 ZZ}}{s - m_{H_k^0}^2} \left( J_{ZZ}^{(5)(H_k^0-\chi_l^0)} \mathcal{B}_{20}(H_k^0, \chi_l^0, Z, Z) + J_{ZZ}^{(6)(H_k^0-\chi_l^0)} \right. \\ &\quad \mathcal{B}_{21}(H_k^0, \chi_l^0, Z, Z) + J_{ZZ}^{(7)(H_k^0-\chi_l^0)} \mathcal{B}_{22}(H_k^0, \chi_l^0, Z, Z) + J_{ZZ}^{(8)(H_k^0-\chi_l^0)} \\ &\quad \left. \mathcal{B}_{23}(H_k^0, \chi_l^0, Z, W) \right) \quad (3.108)\end{aligned}$$

$$\tilde{w}_{ZZ}^{(H_k^0, \chi_l^0)} = \left( \tilde{w}_{ZZ}^{st(H_k^0, \chi_l^0)} + \tilde{w}_{ZZ}^{su(H_k^0, \chi_l^0)} \right) + c.c. \quad (3.109)$$

### 3.11 $\chi_i^0 \chi_j^0 \rightarrow f_m \bar{f}_n$

This process is sum of s channel diagrams through CP even Higgs boson, CP odd Higgs boson and Z boson and sfermion t and u channel. This is the only process which involves flavour violation effects from SUSY soft sector. The resultant cross section is given by:

$$\begin{aligned}\tilde{w}_{f_m \bar{f}_n} &= \sum_{k,l=1,2} \tilde{w}_{f_m \bar{f}_n}^{(H_k^0, H_l^0)} + \tilde{w}_{f_m \bar{f}_n}^{(A_1^0, A_1^0)} + \tilde{w}_{f_m \bar{f}_n}^{(Z, Z)} + \sum_{k=1,2} \tilde{w}_{f_m \bar{f}_n}^{(H_k^0, Z)} + \tilde{w}_{f_m \bar{f}_n}^{(A_1^0, Z)} + \sum_{k,l=1,p} \tilde{w}_{f_m \bar{f}_n}^{(\tilde{f}_k - \tilde{f}_l)} \\ &\quad + \sum_{k=1,2} \sum_{l=1,p} \tilde{w}_{f_m \bar{f}_n}^{(H_k^0 - \tilde{f}_l)} + \sum_{l=1,p} \tilde{w}_{f_m \bar{f}_n}^{(A_1^0 - \tilde{f}_l)} + \sum_{l=1,p} \tilde{w}_{f_m \bar{f}_n}^{(Z - \tilde{f}_l)} \quad (3.110)\end{aligned}$$

- $H_k^0 - H_l^0$ ,  $A_1^0$  and Z boson exchange exchange:

$$\begin{aligned}\tilde{w}_{f_m \bar{f}_n}^{(H_k^0, H_l^0)} &= 2 \frac{C_{H_k^0 f_m \bar{f}_n} C_{H_l^0 f_m \bar{f}_n}^*}{(s - m_{H_k^0}^2)(s - m_{H_l^0}^2)} \left( d f_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} - g_{\chi_i^0 \chi_j^0}^{(H_k^0 - H_l^0)} m_i m_j \right) \\ &\quad (d_{f_m \bar{f}_n} - m_{f_m} m_{\bar{f}_n}) \quad (3.111)\end{aligned}$$

$$\begin{aligned}\tilde{w}_{f_m \bar{f}_n}^{(A_1^0, A_1^0)} &= -2 \frac{C_{A_1^0 f_m \bar{f}_n} C_{A_1^0 f_m \bar{f}_n}^*}{(s - m_{A_1^0}^2)(s - m_{A_1^0}^2)} \left( d f_{\chi_i^0 \chi_j^0}^{(A_1^0 - A_1^0)} - g_{\chi_i^0 \chi_j^0}^{(A_1^0 - A_1^0)} m_i m_j \right) \\ &\quad (d_{f_m \bar{f}_n} + m_{f_m} m_{\bar{f}_n}) \quad (3.112)\end{aligned}$$



$$\begin{aligned}
\tilde{w}_{f_m \bar{f}_n}^{(Z,Z)} = & \frac{|C_{Z f_m \bar{f}_n}^V|^2 + |C_{Z f_m \bar{f}_n}^A|^2}{12 M_Z^4 s^2 (s - M_Z^2)^2} (12 s^2 (2 M_Z^2 - s) (l_{f_m \bar{f}_n} l_{\bar{f}_n f_m} - 2 d_{f_m \bar{f}_n} s) \\
& (d f_{\chi_i^0 \chi_j^0}^{(Z-Z)} + g_{\chi_i^0 \chi_j^0}^{(Z-Z)} m_i m_j) + f_{\chi_i^0 \chi_j^0}^{(Z-Z)} (2 b_{f_m \bar{f}_n}^2 M_Z^4 + 6 l_i l_{ia} s^2 (l_{f_m \bar{f}_n} l_{\bar{f}_n f_m} \\
& + 4 d_{f_m \bar{f}_n} M_Z^2 - 2 d_{f_m \bar{f}_n} s) + 3 (\tau_{f_n f_m} \omega_{f_m \bar{f}_n} + \omega_{f_n f_m} \tau_{f_m \bar{f}_n}) (M_Z^4 - 2 M_Z^2 s)) \\
& + 48 g_{\chi_i^0 \chi_j^0}^{(Z-Z)} s^2 M_Z^4 d_{f_m \bar{f}_n} m_i m_j) + \frac{m_{f_m} m_{\bar{f}_n} (|C_{Z f_m \bar{f}_n}^V|^2 - |C_{Z f_m \bar{f}_n}^A|^2)}{M_Z^4 (s - M_Z^2)^2} \\
& \left( (d f_{\chi_i^0 \chi_j^0}^{(Z-Z)} + g_{\chi_i^0 \chi_j^0}^{(Z-Z)} m_i m_j) (-4 M_Z^2 s + 2 s^2) + 4 M_Z^4 (d f_{\chi_i^0 \chi_j^0}^{(Z-Z)} \right. \\
& + 2 g_{\chi_i^0 \chi_j^0}^{(Z-Z)} m_i m_j) + f_{\chi_i^0 \chi_j^0}^{(Z-Z)} l_i l_{ia} (2 M_Z^2 + s) \Big) \\
& - \frac{(\tau_{f_m \bar{f}_n} \omega_{\bar{f}_n f_m} - \tau_{\bar{f}_n f_m} \omega_{f_m \bar{f}_n}) (s + M_Z^2)}{4 s^2 (s - M_Z^2)^2 M_Z^2} \\
& \left( C_{\chi_i^0 \chi_j^0 Z}^L C_{\chi_i^0 \chi_j^0 Z}^{L*} - C_{\chi_i^0 \chi_j^0 Z}^R C_{\chi_i^0 \chi_j^0 Z}^{R*} \right) \left( C_{Z f_m \bar{f}_n}^{A*} C_{Z f_m \bar{f}_n}^V + C_{Z f_m \bar{f}_n}^A C_{Z f_m \bar{f}_n}^{V*} \right)
\end{aligned} \tag{3.113}$$

$$\tilde{w}_{f_m \bar{f}_n}^{(H_k^0, Z)} = \frac{(l_{\bar{f}_n f_m} m_{f_m} - l_{f_m \bar{f}_n} m_{\bar{f}_n})}{2 M_Z^2 s (s - m_{H_k^0}^2)} C_{Z f_m \bar{f}_n}^{V*} \left( f_{\chi_i^0 \chi_j^0}^{(Z-Z)} l_i m_j - g_{\chi_i^0 \chi_j^0}^{(Z-Z)} l_{ia} m_i \right) \tag{3.114}$$

$$\tilde{w}_{f_m \bar{f}_n}^{(A_1^0, Z)} = - \frac{(l_{\bar{f}_n f_m} m_{f_m} + l_{f_m \bar{f}_n} m_{\bar{f}_n})}{2 M_Z^2 s (s - m_{H_k^0}^2)} C_{Z f_m \bar{f}_n}^{A*} \left( f_{\chi_i^0 \chi_j^0}^{(Z-Z)} l_i m_j - g_{\chi_i^0 \chi_j^0}^{(Z-Z)} l_{ia} m_i \right) \tag{3.115}$$

- Sfermion ( $\tilde{f}_k - \tilde{f}_l$ ) exchange:

$$\begin{aligned}
\tilde{w}_{f_m \bar{f}_n}^{t(\tilde{f}_k - \tilde{f}_l)} = & \frac{1}{8 b_{f_m \bar{f}_n} (R_{f_m \tilde{f}_k} - R_{f_m \tilde{f}_l})} \left( 2 b_{f_m \bar{f}_n} (R_{f_m \tilde{f}_k} - R_{f_m \tilde{f}_l}) \right. \\
& f_{\chi_i^0 f_m}^{(\tilde{f}_k - \tilde{f}_l)} f_{\chi_j^0 \bar{f}_n}^{(\tilde{f}_k - \tilde{f}_l)} \Big) - \frac{\mathcal{X}_1(\tilde{f}_k, \tilde{f}_l, f_m, \bar{f}_n)}{2 b_{f_m \bar{f}_n} (R_{f_m \tilde{f}_k} - R_{f_m \tilde{f}_l})} \left( g_{\chi_i^0 f_m}^{(\tilde{f}_k - \tilde{f}_l)} m_i m_{f_m} s \right. \\
& (f_{\chi_j^0 \bar{f}_n}^{(\tilde{f}_k - \tilde{f}_l)} (\omega_{\bar{f}_n f_m} - \tau_{f_m \bar{f}_n}) + 4 g_{\chi_j^0 \bar{f}_n}^{(\tilde{f}_k - \tilde{f}_l)} m_j m_{\bar{f}_n} s) \Big) \\
& + \frac{\mathcal{Y}_1(\tilde{f}_k, \tilde{f}_l, f_m, \bar{f}_n)}{8 b_{f_m \bar{f}_n} (R_{f_m \tilde{f}_k} - R_{f_m \tilde{f}_l})} \left( f_{\chi_i^0 f_m}^{(\tilde{f}_k - \tilde{f}_l)} f_{\chi_j^0 \bar{f}_n}^{(\tilde{f}_k - \tilde{f}_l)} \right. \\
& (\tau_{f_m \bar{f}_n} - \omega_{\bar{f}_n f_m}) - 4 f_{\chi_j^0 \bar{f}_n}^{(\tilde{f}_k - \tilde{f}_l)} g_{\chi_i^0 f_m}^{(\tilde{f}_k - \tilde{f}_l)} m_i m_{f_m} s - 4 f_{\chi_i^0 f_m}^{(\tilde{f}_k - \tilde{f}_l)} g_{\chi_j^0 \bar{f}_n}^{(\tilde{f}_k - \tilde{f}_l)} \\
& m_j m_{\bar{f}_n} s) - \frac{\mathcal{Z}_1(\tilde{f}_k, \tilde{f}_l, f_m, \bar{f}_n)}{8 b_{f_m \bar{f}_n} (R_{f_m \tilde{f}_k} - R_{f_m \tilde{f}_l})} \left( f_{\chi_i^0 f_m}^{(\tilde{f}_k - \tilde{f}_l)} f_{\chi_j^0 \bar{f}_n}^{(\tilde{f}_k - \tilde{f}_l)} \right)
\end{aligned} \tag{3.116}$$

$$\begin{aligned}
\tilde{w}_{f_m \bar{f}_n}^{u(\tilde{f}_k - \tilde{f}_l)} &= \frac{1}{8 b_{f_m \bar{f}_n} (R_{\bar{f}_n \tilde{f}_k} - R_{\bar{f}_n \tilde{f}_l})} \left( 2 b_{f_m \bar{f}_n} (R_{\bar{f}_n \tilde{f}_k} - R_{\bar{f}_n \tilde{f}_l}) f_{\chi_i^0 \bar{f}_n}^{(\tilde{f}_k - \tilde{f}_l)} f_{\chi_j^0 f_m}^{(\tilde{f}_k - \tilde{f}_l)} \right) \\
&+ \frac{\mathcal{X}_1(\tilde{f}_k, \tilde{f}_l, \bar{f}_n, f_m)}{2 b_{f_m \bar{f}_n} (R_{\bar{f}_n \tilde{f}_k} - R_{\bar{f}_n \tilde{f}_l})} \left( g_{\chi_i^0 \bar{f}_n}^{(\tilde{f}_k - \tilde{f}_l)} m_i m_{\bar{f}_n} s (f_{\chi_j^0 f_m}^{(\tilde{f}_k - \tilde{f}_l)} (\tau_{\bar{f}_n f_m} - \omega_{f_m \bar{f}_n}) \right. \\
&+ 4 g_{\chi_j^0 f_m}^{(\tilde{f}_k - \tilde{f}_l)} m_j m_{f_m} s) \Big) + \frac{\mathcal{Y}_1(\tilde{f}_k, \tilde{f}_l, \bar{f}_n, f_m)}{8 b_{f_m \bar{f}_n} (R_{\bar{f}_n \tilde{f}_k} - R_{\bar{f}_n \tilde{f}_l})} \left( f_{\chi_i^0 \bar{f}_n}^{(\tilde{f}_k - \tilde{f}_l)} f_{\chi_j^0 f_m}^{(\tilde{f}_k - \tilde{f}_l)} \right. \\
&(\omega_{f_m \bar{f}_n} - \tau_{\bar{f}_n f_m}) - 4 f_{\chi_j^0 f_m}^{(\tilde{f}_k - \tilde{f}_l)} g_{\chi_i^0 \bar{f}_n}^{(\tilde{f}_k - \tilde{f}_l)} m_i m_{\bar{f}_n} s + 4 f_{\chi_i^0 \bar{f}_n}^{(\tilde{f}_k - \tilde{f}_l)} g_{\chi_j^0 f_m}^{(\tilde{f}_k - \tilde{f}_l)} \\
&m_j m_{f_m} s) - \frac{\mathcal{Z}_1(\tilde{f}_k, \tilde{f}_l, \bar{f}_n, f_m)}{8 b_{f_m \bar{f}_n} (R_{\bar{f}_n \tilde{f}_k} - R_{\bar{f}_n \tilde{f}_l})} \left( f_{\chi_i^0 \bar{f}_n}^{(\tilde{f}_k - \tilde{f}_l)} f_{\chi_j^0 f_m}^{(\tilde{f}_k - \tilde{f}_l)} \right) \quad (3.117)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{f_m \bar{f}_n}^{tu(\tilde{f}_k - \tilde{f}_l)} &= -\frac{1}{4} J_{f_m \bar{f}_n}^{(1)(\tilde{f}_k - \tilde{f}_l)} + \frac{J_{f_m \bar{f}_n}^{(1)(\tilde{f}_k - \tilde{f}_l)}}{16 b_{f_m \bar{f}_n} (R_{f_m \tilde{f}_k} + R_{\bar{f}_n \tilde{f}_l} - 2 s l_i)} \left( \mathcal{Y}_2(\tilde{f}_k, \tilde{f}_l, f_m, \bar{f}_n) \right. \\
&(\omega_{\bar{f}_n f_m} - \omega_{f_m \bar{f}_n} - (3 R_{f_m \tilde{f}_k} L(\tilde{f}_k, f_m, \bar{f}_n) + R_{\bar{f}_n \tilde{f}_l} L(\tilde{f}_l, \bar{f}_n, f_m)) \tau_{f_m \bar{f}_n}) \\
&- (R_{f_m \tilde{f}_k} L(\tilde{f}_k, f_m, \bar{f}_n) + 3 R_{\bar{f}_n \tilde{f}_l} L(\tilde{f}_l, \bar{f}_n, f_m)) \tau_{\bar{f}_n f_m}) + 2 (R_{f_m \tilde{f}_k}^2 \\
&L(\tilde{f}_k, f_m, \bar{f}_n) + R_{\bar{f}_n \tilde{f}_l}^2 L(\tilde{f}_l, \bar{f}_n, f_m)) + 4 l_i s^2 (l_{f_m \bar{f}_n} L(\tilde{f}_k, f_m, \bar{f}_n) \\
&+ l_{\bar{f}_n f_m} L(\tilde{f}_l, \bar{f}_n, f_m)) \Big) + \frac{s (L(\tilde{f}_k, f_m, \bar{f}_n) + L(\tilde{f}_l, \bar{f}_n, f_m))}{4 b_{f_m \bar{f}_n} (R_{f_m \tilde{f}_k} + R_{\bar{f}_n \tilde{f}_l} - 2 s l_i)} \left( 4 s (-d d_{f_m \bar{f}_n} \right. \\
&J_{f_m \bar{f}_n}^{(1)(\tilde{f}_k - \tilde{f}_l)} + d_{f_m \bar{f}_n} J_{f_m \bar{f}_n}^{(4)(\tilde{f}_k - \tilde{f}_l)} m_i m_j + d J_{f_m \bar{f}_n}^{(2)(\tilde{f}_k - \tilde{f}_l)} m_{f_m} m_{\bar{f}_n} + J_{f_m \bar{f}_n}^{(3)(\tilde{f}_k - \tilde{f}_l)} \\
&m_i m_j m_{f_m} m_{\bar{f}_n}) - \frac{s}{4 b_{f_m \bar{f}_n} (R_{f_m \tilde{f}_k} + R_{\bar{f}_n \tilde{f}_l} - 2 s l_i)} \left( \mathcal{Y}_2(\tilde{f}_k, \tilde{f}_l, f_m, \bar{f}_n) \right. \\
&- \tau_{f_m \bar{f}_n} L(\tilde{f}_k, f_m, \bar{f}_n) + \tau_{\bar{f}_n f_m} L(\tilde{f}_l, \bar{f}_n, f_m) \Big) \left( -J_{f_m \bar{f}_n}^{(8)(\tilde{f}_k - \tilde{f}_l)} m_i m_{f_m} \right. \\
&+ J_{f_m \bar{f}_n}^{(5)(\tilde{f}_k - \tilde{f}_l)} m_j m_{f_m} + J_{f_m \bar{f}_n}^{(7)(\tilde{f}_k - \tilde{f}_l)} m_i m_{\bar{f}_n} - J_{f_m \bar{f}_n}^{(6)(\tilde{f}_k - \tilde{f}_l)} m_j m_{\bar{f}_n} \Big) \Big) \quad (3.118)
\end{aligned}$$

$$\tilde{w}_{f_m \bar{f}_n}^{(\tilde{f}_k - \tilde{f}_l)} = \tilde{w}_{f_m \bar{f}_n}^{t(\tilde{f}_k - \tilde{f}_l)} + \tilde{w}_{f_m \bar{f}_n}^{u(\tilde{f}_k - \tilde{f}_l)} - (\tilde{w}_{f_m \bar{f}_n}^{tu(\tilde{f}_k - \tilde{f}_l)} + c.c) \quad (3.119)$$

- Higgs boson ( $H_k^0, A_1^0$ ), Z boson and sfermion ( $\tilde{f}_l$ ) exchange:

$$\begin{aligned}
\tilde{w}_{f_m \bar{f}_n}^{st(H_k^0 - \tilde{f}_l)} &= -\frac{C_{H_k^0 f_m \bar{f}_n}}{16 b_{f_m \bar{f}_n} (s - m_{H_k^0}^2)} \left( J_{f_m \bar{f}_n}^{(5)(H_k^0 - \tilde{f}_l)} (4 b_{f_m \bar{f}_n} s + (\tau_{f_m \bar{f}_n} + \omega_{f_m \bar{f}_n}) l_i \right. \\
&L(\tilde{f}_l, f_m, \bar{f}_n) - 2 s (4 d (d_{f_m \bar{f}_n} - m_{f_m} m_{\bar{f}_n}) + R_{f_m \tilde{f}_l}) L(\tilde{f}_l, f_m, \bar{f}_n)) \\
&- 8 s m_i m_j L(\tilde{f}_l, f_m, \bar{f}_n) J_{f_m \bar{f}_n}^{(6)(H_k^0 - \tilde{f}_l)} (d_{f_m \bar{f}_n} - m_{f_m} m_{\bar{f}_n}) - 2 m_j \\
&J_{f_m \bar{f}_n}^{(7)(H_k^0 - \tilde{f}_l)} (2 b_{f_m \bar{f}_n} (m_{f_m} + m_{\bar{f}_n}) - ((m_{f_m} + m_{\bar{f}_n}) R_{f_m \tilde{f}_l} - 2 s l_i m_{f_m}) \\
&L(\tilde{f}_l, f_m, \bar{f}_n)) - 2 m_i J_{f_m \bar{f}_n}^{(8)(H_k^0 - \tilde{f}_l)} (2 b_{f_m \bar{f}_n} (m_{f_m} + m_{\bar{f}_n}) - ((m_{f_m} + m_{\bar{f}_n}) \\
&+ (m_{f_m} (\tau_{f_m \bar{f}_n} - \omega_{\bar{f}_n f_m}) - (m_{f_m} + m_{\bar{f}_n}) R_{f_m \tilde{f}_l}) + 2 s m_{\bar{f}_n} l_{f_m \bar{f}_n}) \\
&L(\tilde{f}_l, f_m, \bar{f}_n)) \Big) \quad (3.120)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{f_m \bar{f}_n}^{su(H_k^0 - \tilde{f}_l)} = & -\frac{C_{H_k^0 f_m \bar{f}_n}}{16 b_{f_m \bar{f}_n} (s - m_{H_k^0}^2)} \left( J_{\bar{f}_n f_m}^{(5)(H_k^0 - \tilde{f}_l)} (4 b_{f_m \bar{f}_n} s + (\tau_{\bar{f}_n f_m} + \omega_{\bar{f}_n f_m}) l_i \right. \\
& L(\tilde{f}_l, \bar{f}_n, f_m) - 2 s (d_{f_m \bar{f}_n} - m_{f_m} m_{\bar{f}_n}) + R_{\bar{f}_n \tilde{f}_l}) L(\tilde{f}_l, \bar{f}_n, f_m) \Big) \\
& - 8 s m_i m_j L(\tilde{f}_l, \bar{f}_n, f_m) J_{\bar{f}_n f_m}^{(6)(H_k^0 - \tilde{f}_l)} (d_{f_m \bar{f}_n} - m_{f_m} m_{\bar{f}_n}) - 2 m_j \\
& J_{\bar{f}_n f_m}^{(7)(H_k^0 - \tilde{f}_l)} \left( 2 b_{f_m \bar{f}_n} (m_{f_m} + m_{\bar{f}_n}) - ((m_{f_m} + m_{\bar{f}_n}) R_{\bar{f}_n \tilde{f}_l} - 2 s l_i m_{\bar{f}_n}) \right. \\
& L(\tilde{f}_l, \bar{f}_n, f_m) \Big) - 2 m_i J_{\bar{f}_n f_m}^{(8)(H_k^0 - \tilde{f}_l)} \left( 2 b_{f_m \bar{f}_n} (m_{f_m} + m_{\bar{f}_n}) - ((m_{f_m} + m_{\bar{f}_n}) \right. \\
& + (m_{\bar{f}_n} (\tau_{\bar{f}_n f_m} - \omega_{f_m \bar{f}_n}) - (m_{f_m} + m_{\bar{f}_n}) R_{\bar{f}_n \tilde{f}_l}) + 2 s m_{f_m} l_{\bar{f}_n f_m}) \\
& \left. \left. L(\tilde{f}_l, \bar{f}_n, f_m) \right) \right) \Big) \quad (3.121)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{f_m \bar{f}_n}^{st(A_1^0 - \tilde{f}_l)} = & \frac{C_{A_1^0 f_m \bar{f}_n}}{8 b_{f_m \bar{f}_n} (s - m_{A_1^0}^2)} \left( -s J_{f_m \bar{f}_n}^{(9)(A_1^0 - \tilde{f}_l)} (-2 b_{f_m \bar{f}_n} + (-l_{f_m \bar{f}_n} l_i \right. \\
& + 4 d (d_{f_m \bar{f}_n} + m_{f_m} m_{\bar{f}_n}) + R_{f_m \tilde{f}_l}) L(\tilde{f}_l, f_m, \bar{f}_n) \Big) + 4 m_i m_j s \\
& J_{f_m \bar{f}_n}^{(10)(A_1^0 - \tilde{f}_l)} L(\tilde{f}_l, f_m, \bar{f}_n) (d_{f_m \bar{f}_n} + m_{f_m} m_{\bar{f}_n}) - m_j J_{f_m \bar{f}_n}^{(11)(A_1^0 - \tilde{f}_l)} \\
& \left( (m_{f_m} - m_{\bar{f}_n}) (2 b_{f_m \bar{f}_n} - R_{f_m \tilde{f}_l} L(\tilde{f}_l, f_m, \bar{f}_n)) + 2 l_i m_{f_m} s L(\tilde{f}_l, f_m, \bar{f}_n) \right) \\
& - m_i J_{f_m \bar{f}_n}^{(12)(A_1^0 - \tilde{f}_l)} \left( 2 b_{f_m \bar{f}_n} (m_{f_m} - m_{\bar{f}_n}) + L(\tilde{f}_l, f_m, \bar{f}_n) (m_{f_m} \right. \\
& \left. (\tau_{f_m \bar{f}_n} - \omega_{\bar{f}_n f_m}) - R_{f_m \tilde{f}_l} (m_{f_m} - m_{\bar{f}_n}) - 2 l_{f_m \bar{f}_n} m_{\bar{f}_n} s) \right) \Big) \quad (3.122)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{f_m \bar{f}_n}^{su(A_1^0 - \tilde{f}_l)} = & \frac{C_{A_1^0 f_m \bar{f}_n}}{8 b_{f_m \bar{f}_n} (s - m_{A_1^0}^2)} \left( -s J_{\bar{f}_n f_m}^{(9)(A_1^0 - \tilde{f}_l)} (-2 b_{f_m \bar{f}_n} + (-l_{\bar{f}_n f_m} l_i \right. \\
& + 4 d (d_{f_m \bar{f}_n} + m_{f_m} m_{\bar{f}_n}) + R_{\bar{f}_n \tilde{f}_l}) L(\tilde{f}_l, \bar{f}_n, f_m) \Big) + 4 m_i m_j s \\
& J_{\bar{f}_n f_m}^{(10)(A_1^0 - \tilde{f}_l)} L(\tilde{f}_l, \bar{f}_n, f_m) (d_{f_m \bar{f}_n} + m_{f_m} m_{\bar{f}_n}) - m_j J_{\bar{f}_n f_m}^{(11)(A_1^0 - \tilde{f}_l)} \\
& \left( (m_{\bar{f}_n} - m_{f_m}) (2 b_{f_m \bar{f}_n} - R_{\bar{f}_n \tilde{f}_l} L(\tilde{f}_l, \bar{f}_n, f_m)) + 2 l_i m_{\bar{f}_n} s L(\tilde{f}_l, \bar{f}_n, f_m) \right) \\
& - m_i J_{\bar{f}_n f_m}^{(12)(A_1^0 - \tilde{f}_l)} \left( 2 b_{f_m \bar{f}_n} (m_{\bar{f}_n} - m_{f_m}) + L(\tilde{f}_l, \bar{f}_n, f_m) (m_{\bar{f}_n} \right. \\
& \left. (\tau_{\bar{f}_n f_m} - \omega_{f_m \bar{f}_n}) - R_{\bar{f}_n \tilde{f}_l} (m_{\bar{f}_n} - m_{f_m}) - 2 s l_{\bar{f}_n f_m} m_{f_m}) \right) \Big) \quad (3.123)
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_{f_m \bar{f}_n}^{st(Z-\tilde{f}_l)} = & \frac{C_{Zf_m \bar{f}_n}^V J_{f_m \bar{f}_n}^{(5)(Z-\tilde{f}_l)} - C_{Zf_m \bar{f}_n}^A J_{f_m \bar{f}_n}^{(9)(Z-\tilde{f}_l)}}{8 b_{f_m \bar{f}_n} M_Z^2 (s - M_Z^2)} m_j m_{\bar{f}_n} (s - 4 M_Z^2) (2 b_{f_m \bar{f}_n} \\
& - R_{f_m \tilde{f}_l} L(\tilde{f}_l, f_m, \bar{f}_n)) + \frac{C_{Zf_m \bar{f}_n}^V J_{f_m \bar{f}_n}^{(5)(Z-\tilde{f}_l)} + C_{Zf_m \bar{f}_n}^A J_{f_m \bar{f}_n}^{(9)(Z-\tilde{f}_l)}}{8 b_{f_m \bar{f}_n} M_Z^2 (s - M_Z^2)} \\
& m_j m_{f_m} \left( (s - 2 M_Z^2) ((R_{f_m \tilde{f}_l} - 2 l_i s) L(\tilde{f}_l, f_m, \bar{f}_n) - 2 b_{f_m \bar{f}_n}) \right. \\
& \left. + 2 s \tau_{\bar{f}_n f_m} L(\tilde{f}_l, f_m, \bar{f}_n) \right) + \frac{C_{Zf_m \bar{f}_n}^V J_{f_m \bar{f}_n}^{(6)(Z-\tilde{f}_l)} - C_{Zf_m \bar{f}_n}^A J_{f_m \bar{f}_n}^{(10)(Z-\tilde{f}_l)}}{8 b_{f_m \bar{f}_n} M_Z^2 (s - M_Z^2)} \\
& m_i m_{\bar{f}_n} \left( (s - 2 M_Z^2) ((R_{f_m \tilde{f}_l} - 2 l_{f_m \bar{f}_n} s) L(\tilde{f}_l, f_m, \bar{f}_n) - 2 b_{f_m \bar{f}_n}) \right. \\
& \left. + 2 s \omega_{f_m \bar{f}_n} L(\tilde{f}_l, f_m, \bar{f}_n) \right) + \frac{C_{Zf_m \bar{f}_n}^V J_{f_m \bar{f}_n}^{(6)(Z-\tilde{f}_l)} + C_{Zf_m \bar{f}_n}^A J_{f_m \bar{f}_n}^{(10)(Z-\tilde{f}_l)}}{8 b_{f_m \bar{f}_n} M_Z^2 (s - M_Z^2)} \\
& m_i m_{f_m} (s - 4 M_Z^2) \left( (\tau_{f_m \bar{f}_n} - \omega_{\bar{f}_n f_m} - R_{f_m \tilde{f}_l}) L(\tilde{f}_l, f_m, \bar{f}_n) + 2 b_{f_m \bar{f}_n} \right) \\
& - \frac{C_{Zf_m \bar{f}_n}^V J_{f_m \bar{f}_n}^{(7)(Z-\tilde{f}_l)} - C_{Zf_m \bar{f}_n}^A J_{f_m \bar{f}_n}^{(11)(Z-\tilde{f}_l)}}{32 s b_{f_m \bar{f}_n} M_Z^2 (s - M_Z^2)} (8 b_{f_m \bar{f}_n} M_Z^2 \\
& (\omega_{\bar{f}_n f_m} + R_{f_m \tilde{f}_l}) + 2 b_{f_m \bar{f}_n} s (\tau_{\bar{f}_n f_m} - \tau_{f_m \bar{f}_n} + \omega_{f_m \bar{f}_n} - \omega_{\bar{f}_n f_m}) \\
& - (4 M_Z^2 R_{f_m \tilde{f}_l} (R_{f_m \tilde{f}_l} - \tau_{f_m \bar{f}_n}) + \tau_{f_m \bar{f}_n}^2 s - 3 \tau_{\bar{f}_n f_m} \omega_{f_m \bar{f}_n} s \\
& + s \tau_{\bar{f}_n f_m} R_{f_m \tilde{f}_l} + \tau_{f_m \bar{f}_n} R_{f_m \tilde{f}_l} s - \tau_{f_m \bar{f}_n} (\tau_{\bar{f}_n f_m} + \omega_{f_m \bar{f}_n} + R_{f_m \tilde{f}_l}) s \\
& - 16 d d_{f_m \bar{f}_n} s^3 - \omega_{\bar{f}_n f_m} R_{f_m \tilde{f}_l} (s - 4 M_Z^2) + 8 d_{f_m \bar{f}_n} s^2 l_i l_{ia} \\
& + 8 d s^2 l_{f_m \bar{f}_n} l_{\bar{f}_n f_m}) L(\tilde{f}_l, f_m, \bar{f}_n) \\
& + \frac{C_{Zf_m \bar{f}_n}^V J_{f_m \bar{f}_n}^{(7)(Z-\tilde{f}_l)} + C_{Zf_m \bar{f}_n}^A J_{f_m \bar{f}_n}^{(11)(Z-\tilde{f}_l)}}{4 b_{f_m \bar{f}_n} M_Z^2} m_{f_m} m_{\bar{f}_n} s L(\tilde{f}_l, f_m, \bar{f}_n) \\
& (l_i l_{ia} - 2 d (s - 2 M_Z^2)) + \frac{C_{Zf_m \bar{f}_n}^V J_{f_m \bar{f}_n}^{(8)(Z-\tilde{f}_l)} - C_{Zf_m \bar{f}_n}^A J_{f_m \bar{f}_n}^{(12)(Z-\tilde{f}_l)}}{4 b_{f_m \bar{f}_n} M_Z^2} \\
& (l_{f_m \bar{f}_n} l_{\bar{f}_n f_m} - 2 d_{f_m \bar{f}_n} (s - 2 M_Z^2)) \\
& + \frac{C_{Zf_m \bar{f}_n}^V J_{f_m \bar{f}_n}^{(8)(Z-\tilde{f}_l)} + C_{Zf_m \bar{f}_n}^A J_{f_m \bar{f}_n}^{(12)(Z-\tilde{f}_l)}}{2 b_{f_m \bar{f}_n} M_Z^2} m_i m_j m_{f_m} m_{\bar{f}_n} (s - 4 M_Z^2) \\
& L(\tilde{f}_l, \bar{f}_n, f_m)
\end{aligned} \tag{3.124}$$

$$\begin{aligned}
\tilde{w}_{f_m \bar{f}_n}^{su(Z-\tilde{f}_l)} = & \frac{C_{Z f_m \bar{f}_n}^V J_{\bar{f}_n f_m}^{(5)(Z-\tilde{f}_l)} + C_{Z f_m \bar{f}_n}^A J_{\bar{f}_n f_m}^{(9)(Z-\tilde{f}_l)}}{8 b_{f_m \bar{f}_n} M_Z^2 (s - M_Z^2)} m_j m_{f_m} (s - 4 M_Z^2) (2 b_{f_m \bar{f}_n} \\
& - R_{\bar{f}_n \tilde{f}_l} L(\tilde{f}_l, \bar{f}_n, f_m)) + \frac{C_{Z f_m \bar{f}_n}^V J_{\bar{f}_n f_m}^{(5)(Z-\tilde{f}_l)} - C_{Z f_m \bar{f}_n}^A J_{\bar{f}_n f_m}^{(9)(Z-\tilde{f}_l)}}{8 b_{f_m \bar{f}_n} M_Z^2 (s - M_Z^2)} \\
& m_j m_{\bar{f}_n} \left( (s - 2 M_Z^2) ((R_{\bar{f}_n \tilde{f}_l} - 2 l_i s) L(\tilde{f}_l, \bar{f}_n, f_m) - 2 b_{f_m \bar{f}_n}) \right. \\
& \left. + 2 s \tau_{f_m \bar{f}_n} L(\tilde{f}_l, \bar{f}_n, f_m) \right) + \frac{C_{Z f_m \bar{f}_n}^V J_{\bar{f}_n f_m}^{(6)(Z-\tilde{f}_l)} + C_{Z f_m \bar{f}_n}^A J_{\bar{f}_n f_m}^{(10)(Z-\tilde{f}_l)}}{8 b_{f_m \bar{f}_n} M_Z^2 (s - M_Z^2)} \\
& m_i m_{f_m} \left( (s - 2 M_Z^2) ((R_{\bar{f}_n \tilde{f}_l} - 2 l_{\bar{f}_n f_m} s) L(\tilde{f}_l, \bar{f}_n, f_m) - 2 b_{f_m \bar{f}_n}) \right. \\
& \left. + 2 s \omega_{\bar{f}_n f_m} L(\tilde{f}_l, \bar{f}_n, f_m) \right) + \frac{C_{Z f_m \bar{f}_n}^V J_{\bar{f}_n f_m}^{(6)(Z-\tilde{f}_l)} - C_{Z f_m \bar{f}_n}^A J_{\bar{f}_n f_m}^{(10)(Z-\tilde{f}_l)}}{8 b_{f_m \bar{f}_n} M_Z^2 (s - M_Z^2)} \\
& m_i m_{\bar{f}_n} (s - 4 M_Z^2) \left( (\tau_{\bar{f}_n f_m} - \omega_{f_m \bar{f}_n} - R_{\bar{f}_n \tilde{f}_l}) L(\tilde{f}_l, \bar{f}_n, f_m) + 2 b_{f_m \bar{f}_n} \right) \\
& - \frac{C_{Z f_m \bar{f}_n}^V J_{\bar{f}_n f_m}^{(7)(Z-\tilde{f}_l)} + C_{Z f_m \bar{f}_n}^A J_{\bar{f}_n f_m}^{(11)(Z-\tilde{f}_l)}}{32 s b_{f_m \bar{f}_n} M_Z^2 (s - M_Z^2)} (8 b_{f_m \bar{f}_n} M_Z^2 \\
& (\omega_{f_m \bar{f}_n} + R_{\bar{f}_n \tilde{f}_l}) + 2 b_{f_m \bar{f}_n} s (\tau_{f_m \bar{f}_n} - \tau_{\bar{f}_n f_m} + \omega_{\bar{f}_n f_m} - \omega_{f_m \bar{f}_n}) \\
& - (4 M_Z^2 R_{\bar{f}_n \tilde{f}_l} (R_{\bar{f}_n \tilde{f}_l} - \tau_{\bar{f}_n f_m}) + \tau_{\bar{f}_n f_m}^2 s - 3 \tau_{f_m \bar{f}_n} \omega_{\bar{f}_n f_m} s \\
& + s \tau_{f_m \bar{f}_n} R_{\bar{f}_n \tilde{f}_l} + \tau_{\bar{f}_n f_m} R_{\bar{f}_n \tilde{f}_l} s - \tau_{\bar{f}_n f_m} (\tau_{f_m \bar{f}_n} + \omega_{\bar{f}_n f_m} + R_{\bar{f}_n \tilde{f}_l}) s \\
& - 16 d d_{f_m \bar{f}_n} s^3 - \omega_{f_m \bar{f}_n} R_{\bar{f}_n \tilde{f}_l} (s - 4 M_Z^2) + 8 d_{f_m \bar{f}_n} s^2 l_i l_{ia} \\
& + 8 d s^2 l_{f_m \bar{f}_n} l_{\bar{f}_n f_m}) L(\tilde{f}_l, \bar{f}_n, f_m) \Big) \\
& + \frac{C_{Z f_m \bar{f}_n}^V J_{\bar{f}_n f_m}^{(7)(Z-\tilde{f}_l)} - C_{Z f_m \bar{f}_n}^A J_{\bar{f}_n f_m}^{(11)(Z-\tilde{f}_l)}}{4 b_{f_m \bar{f}_n} M_Z^2} m_{f_m} m_{\bar{f}_n} s L(\tilde{f}_l, \bar{f}_n, f_m) \\
& (l_i l_{ia} - 2 d (s - 2 M_Z^2)) + \frac{C_{Z f_m \bar{f}_n}^V J_{\bar{f}_n f_m}^{(8)(Z-\tilde{f}_l)} + C_{Z f_m \bar{f}_n}^A J_{\bar{f}_n f_m}^{(12)(Z-\tilde{f}_l)}}{4 b_{f_m \bar{f}_n} M_Z^2} \\
& (l_{f_m \bar{f}_n} l_{\bar{f}_n f_m} - 2 d_{f_m \bar{f}_n} (s - 2 M_Z^2)) \\
& + \frac{C_{Z f_m \bar{f}_n}^V J_{\bar{f}_n f_m}^{(8)(Z-\tilde{f}_l)} - C_{Z f_m \bar{f}_n}^A J_{\bar{f}_n f_m}^{(12)(Z-\tilde{f}_l)}}{2 b_{f_m \bar{f}_n} M_Z^2} m_i m_j m_{f_m} m_{\bar{f}_n} (s - 4 M_Z^2) \\
& L(\tilde{f}_l, \bar{f}_n, f_m)
\end{aligned} \tag{3.125}$$

$$\tilde{w}_{f_m \bar{f}_n}^{(H_k^0 - \tilde{f}_l)} = (\tilde{w}_{f_m \bar{f}_n}^{st(H_k^0 - \tilde{f}_l)} - \tilde{w}_{f_m \bar{f}_n}^{su(H_k^0 - \tilde{f}_l)}) + c.c. \tag{3.126}$$

$$\tilde{w}_{f_m \bar{f}_n}^{(A_1^0 - \tilde{f}_l)} = (\tilde{w}_{f_m \bar{f}_n}^{st(A_1^0 - \tilde{f}_l)} - \tilde{w}_{f_m \bar{f}_n}^{su(A_1^0 - \tilde{f}_l)}) + c.c. \tag{3.127}$$

$$\tilde{w}_{f_m \bar{f}_n}^{(Z - \tilde{f}_l)} = (\tilde{w}_{f_m \bar{f}_n}^{s(Z - \tilde{f}_l)} - \tilde{w}_{f_m \bar{f}_n}^{su(Z - \tilde{f}_l)}) + c.c. \tag{3.128}$$

where  $\tilde{f}_k$  is a sfermion mass eigen stat with  $k=1, \dots, p$ ; for squarks and charged leptons  $p=6$  and for  $s$  neutrinos  $p=3$

## 4 Summary

We have presented, for the first time the full neutralino neutralino annihilation cross section with flavour violation. And we compared our results with the ones given in Nihei et al. We hope to extend this analysis to neutralino-slepton co-annihilation where we expect large changes and also to Sommerfeld enhancement.

## A Mixing matrices and Coupling constants

In this appendix , we collect various couplings involving mixing matrices are listed. Feynman diagrams corresponding to all the annihilation channels are given in [18]. We closely follow Ref.[18] for notational consistency. For the sake of ease and completeness we present in this section relevant spectrum which is used in calculations in this paper. And for more details we urge readers to refer to [18]

- Neutral Higgs scalars. Two CP even and one CP odd Higgs boson eigen values are given in terms of their mixing matrices.

- i) CP even Higgs boson  $H_i^0$  ,  $i = 1, 2$ , defined as:

$$\sqrt{2}\Re H_i^0 = Z_R^{ij} H_j^0 + v_i \text{ (no sum over } i \text{)}$$

- ii) CP odd Higgs boson  $A_i^0$  ,  $i = 1, 2$ :

$$\sqrt{2}\Im H_i^0 = Z_H^{ij} A_j^0$$

$A_1^0$  is the CP odd Higgs boson and  $A_2^0$  is the mass less Goldstone boson which is not a physical degree of freedom.

$$Z_H = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \quad Z_R = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \quad (\text{A.1})$$

where:

$$\tan\beta = \frac{v_2}{v_1} \quad (\text{A.2})$$

$$\tan 2\alpha = \tan 2\beta \frac{m_A^2 + M_Z^2}{m_A^2 - M_Z^2} \quad (\text{A.3})$$

$$m_A^2 = m_{H_1}^2 + m_{H_2}^2 + 2\mu^2 \quad (\text{A.4})$$

$$A_M^{ij} = Z_R^{1i} Z_H^{1j} - Z_R^{2i} Z_H^{2j} \quad A_R^{ij} = Z_R^{1i} Z_R^{1j} - Z_R^{2i} Z_H^{2j} \quad (\text{A.5})$$

$$A_H^{ij} = Z_H^{1i} Z_H^{1j} - Z_H^{2i} Z_H^{2j} \quad B_R^i = v_1 Z_R^{1i} - v_2 Z_R^{2i} \quad (\text{A.6})$$

$$A_P^{ij} = Z_R^{1i} Z_H^{2j} + Z_R^{2i} Z_H^{1j} \quad (\text{A.7})$$

- Charged Higgs scalars: There exist two charged physical Higgs bsons and two charged Goldstone bosons in MSSM. Masses of the physical charged Higgses are given by

$$M_{H_1^\pm}^2 = M_W^2 + m_{H_1}^2 + m_{H_2}^2 + 2|\mu|^2 \quad (\text{A.8})$$

and the charged Goldstone bosons are mass less. In the unitary gauge  $H_2^\pm (\equiv G^\pm)$  disappear as they are not physical degrees of freedom. And  $Z_H$  is matrix which mixes  $H_1^+$  and  $H_2^+$  with the initial Higgs fields.

$$C_{A_i^0 A_j^0 H_k^0} = -\frac{ie^2}{4s_W^2 c_W^2} A_R^{ij} B_R^k \quad (\text{A.9})$$

$$C_{H_i^0 H_j^0 H_k^0} = -\frac{ie^2}{4s_W^2 c_W^2} (A_R^{ij} B_R^k + A_R^{jk} B_R^i + A_R^{ki} B_R^j) \quad (\text{A.10})$$

$$C_{H_i^0 H_1^+ H_1^-} = -i \left( \frac{e^2}{4s_W^2 c_W^2} A_H^{11} B_R^i + \frac{e M_W}{s_W} A_P^{i1} \right) \quad (\text{A.11})$$

$$C_{H_i^0 A_j^0 Z} = \frac{e}{2s_W c_W} A_M^{ij} \quad (\text{A.12})$$

$$C_{H_1^+ H_1^- Z} = ie \frac{c_W^2 - s_W^2}{2s_W c_W} \quad (\text{A.13})$$

$$C_{A_1^0 W^+ H^-} = -\frac{e}{2s_W} \quad (\text{A.14})$$

$$C_{H_i^0 W^+ H^-} = -\frac{e}{2s_W} A_M^{i1} \quad (\text{A.15})$$

$$C_{H_i^0 ZZ} = \frac{ie^2}{2s_W^2 c_W^2} g^{\mu\nu} C_R^i \quad (\text{A.16})$$

$$C_{H_k^0 ZZ} = \frac{ie^2}{2s_W^2 c_W^2} g^{\mu\nu} [v_1 Z_R^{1k} + v_2 Z_R^{2k}] \quad (\text{A.17})$$

$$C_{H_k^0 WW} = \frac{ie^2}{2s_W^2} g^{\mu\nu} [v_1 Z_R^{1k} + v_2 Z_R^{2k}] \quad (\text{A.18})$$

$$C_{Z f_m \bar{f}_n}^V = \frac{ie^2}{4s_W^2 c_W^2} (1 - 2s_W^2) \quad (\text{A.19})$$

$$C_{Z f_m \bar{f}_n}^A = \frac{ie^2}{4s_W^2 c_W^2} \quad (\text{A.20})$$

$$C_{\chi_i^0 \chi_l^0 H_m^0}^L = \frac{e}{2s_W c_W} [(Z_R^{1m} Z_N^{3i} - Z_R^{2m} Z_N^{4i})(Z_N^{1l} s_W - Z_N^{2l} c_W)] \quad (\text{A.21})$$

$$C_{\chi_i^0 \chi_l^0 H_m^0}^R = \frac{e}{2s_W c_W} [(Z_R^{1m} Z_N^{3l*} - Z_R^{2m} Z_N^{4l*})(Z_N^{1i*} s_W - Z_N^{2i*} c_W)] \quad (\text{A.22})$$

$$C_{\chi_i^0 \chi_j^0 A_n^0}^L = \frac{ie}{2s_W c_W} [(Z_H^{1n} Z_N^{3l} - Z_H^{2n} Z_N^{4l})(Z_N^{1j} s_W - Z_N^{2j} c_W)] \quad (\text{A.23})$$

$$C_{\chi_i^0 \chi_j^0 A_n^0}^R = \frac{-ie}{2s_W c_W} [(Z_H^{1n} Z_N^{3j*} - Z_H^{2n} Z_N^{4j*})(Z_N^{1i*} s_W - Z_N^{2i*} c_W)] \quad (\text{A.24})$$

$$C_{\chi_i^0 \chi_j^0 Z}^L = \frac{e}{4s_W c_W} (Z_N^{4j*} Z_N^{4i} - Z_N^{3j*} Z_N^{3i}) P_L \quad (\text{A.25})$$

$$C_{\chi_i^0 \chi_j^0 Z}^R = \frac{-e}{4s_W c_W} (Z_N^{4j} Z_N^{4i*} - Z_N^{3j} Z_N^{3i*}) P_R \quad (\text{A.26})$$

$$C_{\chi_i^0 \chi_l^0 Z}^L = \frac{ie}{2s_W c_W} \left[ (Z_N^{4i*} Z_N^{4l} - Z_N^{3i*} Z_N^{3l}) \right] \quad (\text{A.27})$$

$$C_{\chi_i^0 \chi_l^0 Z}^R = \frac{ie}{2s_W c_W} \left[ (Z_N^{3i} Z_N^{3l*} - Z_N^{4i} Z_N^{4l*}) \right] \quad (\text{A.28})$$

$$C_{\chi_i^0 \chi^j H_k^+}^L = \frac{ie}{s_W c_W} \left[ Z_H^{1k} \left( \frac{1}{\sqrt{2}} Z_-^{2j} (Z_N^{1i} s_W + Z_N^{2i} c_W) - Z_-^{1j} Z_N^{3i} c_W \right) \right] \quad (\text{A.29})$$

$$C_{\chi_i^0 \chi^j H_k^+}^R = -\frac{ie}{s_W c_W} \left[ Z_H^{2k} \left( \frac{1}{\sqrt{2}} Z_+^{2j*} (Z_N^{1i*} s_W + Z_N^{2i*} c_W) + Z_+^{1j*} Z_N^{4i*} c_W \right) \right] \quad (\text{A.30})$$

$$C_{\chi_i^0 \chi_l^\pm W}^L = \frac{ie}{s_W} \left[ (Z_N^{2l} Z_+^{1i*} - \frac{1}{\sqrt{2}} Z_N^{4l} Z_+^{2i*}) \right] \quad (\text{A.31})$$

$$C_{\chi_i^0 \chi_l^\pm W}^R = \frac{ie}{s_W} \left[ (Z_N^{2l*} Z_-^{1i} + \frac{1}{\sqrt{2}} Z_N^{4l*} Z_-^{2i}) \right] \quad (\text{A.32})$$

$$f_{\chi_i^0 \chi_j^0}^{A-B} = C_{\chi_i^0 \chi_j^0 A}^L C_{\chi_i^0 \chi_j^0 B}^{L*} + C_{\chi_i^0 \chi_j^0 A}^R C_{\chi_i^0 \chi_j^0 B}^{R*}$$

$$g_{\chi_i^0 \chi_j^0}^{AB} = C_{\chi_i^0 \chi_j^0 A}^L C_{\chi_i^0 \chi_j^0 B}^{R*} + C_{\chi_i^0 \chi_j^0 A}^R C_{\chi_i^0 \chi_j^0 B}^{L*}$$

$$\Gamma_{\phi_1 \phi_2}^{1(A-B)} = C_{\chi_i^0 A \phi_1}^L C_{\chi_i^0 B \phi_1}^{L*} C_{\chi_j^0 A \phi_2}^L C_{\chi_j^0 B \phi_2}^{L*} + C_{\chi_i^0 A \phi_1}^R C_{\chi_i^0 B \phi_1}^{R*} C_{\chi_j^0 A \phi_2}^R C_{\chi_j^0 B \phi_2}^{R*}$$

$$\Gamma_{\phi_1 \phi_2}^{2(A-B)} = C_{\chi_i^0 A \phi_1}^L C_{\chi_i^0 B \phi_1}^{L*} C_{\chi_j^0 A \phi_2}^R C_{\chi_j^0 B \phi_2}^{R*} + C_{\chi_i^0 A \phi_1}^R C_{\chi_i^0 B \phi_1}^{R*} C_{\chi_j^0 A \phi_2}^L C_{\chi_j^0 B \phi_2}^{L*}$$

$$\Gamma_{\phi_1 \phi_2}^{3(A-B)} = C_{\chi_i^0 A \phi_1}^L C_{\chi_i^0 B \phi_1}^{R*} C_{\chi_j^0 A \phi_2}^L C_{\chi_j^0 B \phi_2}^{R*} + C_{\chi_i^0 A \phi_1}^R C_{\chi_i^0 B \phi_1}^{L*} C_{\chi_j^0 A \phi_2}^R C_{\chi_j^0 B \phi_2}^{L*}$$

$$\Gamma_{\phi_1 \phi_2}^{4(A-B)} = C_{\chi_i^0 A \phi_1}^L C_{\chi_i^0 B \phi_1}^{R*} C_{\chi_j^0 A \phi_2}^R C_{\chi_j^0 B \phi_2}^{L*} + C_{\chi_i^0 A \phi_1}^R C_{\chi_i^0 B \phi_1}^{L*} C_{\chi_j^0 A \phi_2}^L C_{\chi_j^0 B \phi_2}^{R*}$$

$$\Gamma_{\phi_1 \phi_2}^{5(A-B)} = C_{\chi_i^0 A \phi_1}^L C_{\chi_i^0 B \phi_1}^{L*} C_{\chi_j^0 A \phi_2}^L C_{\chi_j^0 B \phi_2}^{R*} + C_{\chi_i^0 A \phi_1}^R C_{\chi_i^0 B \phi_1}^{R*} C_{\chi_j^0 A \phi_2}^R C_{\chi_j^0 B \phi_2}^{L*}$$

$$\Gamma_{\phi_1 \phi_2}^{6(A-B)} = C_{\chi_i^0 A \phi_1}^L C_{\chi_i^0 B \phi_1}^{L*} C_{\chi_j^0 A \phi_2}^R C_{\chi_j^0 B \phi_2}^{L*} + C_{\chi_i^0 A \phi_1}^R C_{\chi_i^0 B \phi_1}^{R*} C_{\chi_j^0 A \phi_2}^L C_{\chi_j^0 B \phi_2}^{R*}$$

$$\Gamma_{\phi_1 \phi_2}^{7(A-B)} = C_{\chi_i^0 A \phi_1}^L C_{\chi_i^0 B \phi_1}^{R*} C_{\chi_j^0 A \phi_2}^L C_{\chi_j^0 B \phi_2}^{L*} + C_{\chi_i^0 A \phi_1}^R C_{\chi_i^0 B \phi_1}^{L*} C_{\chi_j^0 A \phi_2}^R C_{\chi_j^0 B \phi_2}^{R*}$$

$$\Gamma_{\phi_1 \phi_2}^{8(A-B)} = C_{\chi_i^0 A \phi_1}^L C_{\chi_i^0 B \phi_1}^{R*} C_{\chi_j^0 A \phi_2}^R C_{\chi_j^0 B \phi_2}^{R*} + C_{\chi_i^0 A \phi_1}^R C_{\chi_i^0 B \phi_1}^{L*} C_{\chi_j^0 A \phi_2}^L C_{\chi_j^0 B \phi_2}^{L*}$$

$$I_{\phi_1 \phi_2}^{1(A-B)} = C_{\chi_i^0 A \phi_1}^L C_{\chi_i^0 B \phi_2}^{L*} C_{\chi_j^0 A \phi_2}^L C_{\chi_j^0 B \phi_1}^{L*} + C_{\chi_i^0 A \phi_1}^R C_{\chi_i^0 B \phi_2}^{R*} C_{\chi_j^0 A \phi_2}^R C_{\chi_j^0 B \phi_1}^{R*}$$

$$I_{\phi_1 \phi_2}^{2(A-B)} = C_{\chi_i^0 A \phi_1}^L C_{\chi_i^0 B \phi_2}^{L*} C_{\chi_j^0 A \phi_2}^R C_{\chi_j^0 B \phi_1}^{R*} + C_{\chi_i^0 A \phi_1}^R C_{\chi_i^0 B \phi_2}^{R*} C_{\chi_j^0 A \phi_2}^L C_{\chi_j^0 B \phi_1}^{L*}$$

$$I_{\phi_1 \phi_2}^{3(A-B)} = C_{\chi_i^0 A \phi_1}^L C_{\chi_i^0 B \phi_2}^{R*} C_{\chi_j^0 A \phi_2}^L C_{\chi_j^0 B \phi_1}^{R*} + C_{\chi_i^0 A \phi_1}^R C_{\chi_i^0 B \phi_2}^{L*} C_{\chi_j^0 A \phi_2}^R C_{\chi_j^0 B \phi_1}^{L*}$$



[illegible]

## B Auxiliary functions

In this appendix, we collected various kinetic functions and auxiliary functions used in the main text.

$$\begin{aligned}
l_i &= m_i^2 - m_j^2 + s \\
l_{ia} &= m_j^2 - m_i^2 + s \\
l_{\phi_1\phi_2} &= m_1^2 - m_2^2 + s \\
\tau_{\phi_1\phi_2} &= l_i (m_1^2 - m_2^2 + s) \\
\omega_{\phi_1\phi_2} &= l_{ia} (m_1^2 - m_2^2 + s) \\
R_{\phi_1\phi_2} &= 2(m_1^2 - m_2^2 + m_i^2)s \\
b_{\phi_1\phi_2} &= \sqrt{s^2 + m_i^4 + m_j^4 - 2s(m_i^2 + m_j^2) - 2m_i^2 m_j^2} \\
&\quad \sqrt{s^2 + m_1^4 + m_2^4 - 2s(m_1^2 + m_2^2) - 2m_1^2 m_2^2} \\
d &= \frac{1}{2} (s - (m_i^2 + m_j^2)) \\
d_{\phi_1\phi_2} &= \frac{1}{2} (s - (m_1^2 + m_2^2)) \\
L(\chi, \phi_1, \phi_2) &= \log \frac{R_{\phi_1\chi} + b_{\phi_1\phi_2} - \tau_{\phi_1\phi_2}}{R_{\phi_1\chi} - b_{\phi_1\phi_2} - \tau_{\phi_1\phi_2}} \\
\mathcal{A}(\chi, \phi_1, \phi_2) &= \operatorname{arctanh} \frac{b_{\phi_1\phi_2}}{R_{\phi_1\chi} - \tau_{\phi_1\phi_2}} \\
\mathcal{B}(\chi, \phi_1, \phi_2) &= R_{\phi_1\chi} - \tau_{\phi_1\phi_2} - \omega_{\phi_1\phi_2} \\
\mathcal{C}(\chi, \phi_1, \phi_2) &= R_{\phi_1\chi} - \tau_{\phi_1\phi_2} + \omega_{\phi_2\phi_1} \\
\rho(\chi, \phi_1, \phi_2) &= R_{\phi_1\chi} - \omega_{\phi_1\phi_2} \\
\mathcal{X}(\phi_1, \phi_2, \chi_1) &= -8s^2 (2d d_{\phi_1\phi_2} - (2d + l_{ia} m_i^2) + 2R_{\phi_1\chi_1}^2 - R_{\phi_1\chi_1} (3\tau_{\phi_1\phi_2} + \tau_{\phi_2\phi_1} \\
&\quad + \omega_{\phi_1\phi_2} - \omega_{\phi_2\phi_1}) + \tau_{\phi_1\phi_2} (\tau_{\phi_1\phi_2} + \tau_{\phi_2\phi_1} + \omega_{\phi_1\phi_2} + \omega_{\phi_2\phi_1}) \\
\mathcal{Q}_1(\chi_1, \chi_2, \phi_1, \phi_2) &= R_{\phi_1\chi_1}^4 L(\chi_1, \phi_1, \phi_2) - R_{\phi_1\chi_2}^4 L(\chi_2, \phi_1, \phi_2) \\
\mathcal{R}_1(\chi_1, \chi_2, \phi_1, \phi_2) &= R_{\phi_1\chi_1}^3 L(\chi_1, \phi_1, \phi_2) - R_{\phi_1\chi_2}^3 L(\chi_2, \phi_1, \phi_2) \\
\mathcal{Z}_1(\chi_1, \chi_2, \phi_1, \phi_2) &= R_{\phi_1\chi_1}^2 L(\chi_1, \phi_1, \phi_2) - R_{\phi_1\chi_2}^2 L(\chi_2, \phi_1, \phi_2) \\
\mathcal{Y}_1(\chi_1, \chi_2, \phi_1, \phi_2) &= R_{\phi_1\chi_1} L(\chi_1, \phi_1, \phi_2) - R_{\phi_1\chi_2} L(\chi_2, \phi_1, \phi_2) \\
\mathcal{X}_1(\chi_1, \chi_2, \phi_1, \phi_2) &= L(\chi_1, \phi_1, \phi_2) - L(\chi_2, \phi_1, \phi_2) \\
\mathcal{Q}_2(\chi_1, \chi_2, \phi_1, \phi_2) &= R_{\phi_1\chi_1}^4 L(\chi_1, \phi_1, \phi_2) - R_{\phi_2\chi_2}^4 L(\chi_2, \phi_2, \phi_1) \\
\mathcal{R}_2(\chi_1, \chi_2, \phi_1, \phi_2) &= R_{\phi_1\chi_1}^3 L(\chi_1, \phi_1, \phi_2) - R_{\phi_2\chi_2}^3 L(\chi_2, \phi_2, \phi_1) \\
\mathcal{Z}_2(\chi_1, \chi_2, \phi_1, \phi_2) &= R_{\phi_1\chi_1}^2 L(\chi_1, \phi_1, \phi_2) - R_{\phi_2\chi_2}^2 L(\chi_2, \phi_2, \phi_1) \\
\mathcal{Y}_2(\chi_1, \chi_2, \phi_1, \phi_2) &= R_{\phi_1\chi_1} L(\chi_1, \phi_1, \phi_2) - R_{\phi_2\chi_2} L(\chi_2, \phi_2, \phi_1) \\
\mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2) &= L(\chi_1, \phi_1, \phi_2) - L(\chi_2, \phi_2, \phi_1)
\end{aligned}$$

$$\begin{aligned}
\beta_1(\phi_1, \phi_2, \chi_1, \chi_2) &= -4d(m_1^2 - m_i^2) - \frac{1}{4s}(s + 8m_i^2\omega_{\phi_1\phi_2}) - \frac{1}{4b_{\phi_1\phi_2}(R_{\phi_1\chi_1} - R_{\phi_1\chi_2})} \\
&\quad [R_{\phi_1\chi_2}\mathcal{B}(\chi_2, \phi_1, \phi_2)\mathcal{A}(\chi_2, \phi_1, \phi_2) - \{\chi_2 \rightarrow \chi_1\}] \\
\beta_2(\phi_1, \phi_2, \chi_1, \chi_2) &= 4(m_1^2 + m_i^2) + \frac{4s}{b_{\phi_1\phi_2}(R_{\phi_1\chi_1} - R_{\phi_1\chi_2})} \\
&\quad [R_{\phi_1\chi_2}\mathcal{B}(\chi_1, \phi_1, \phi_2)\mathcal{A}(\chi_1, \phi_1, \phi_2) - \{\chi_1 \rightarrow \chi_2\}] \\
\beta_3(\phi_1, \phi_2, \chi_1, \chi_2) &= 4m_i^2 + \frac{s}{2b_{\phi_1\phi_2}(R_{\phi_1\chi_1} - R_{\phi_1\chi_2})} \\
&\quad [R_{\phi_1\chi_2}\mathcal{B}(\chi_1, \phi_1, \phi_2)\mathcal{A}(\chi_1, \phi_1, \phi_2) - \{\chi_1 \rightarrow \chi_2\}] \\
J_1(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{L(\chi_1, \phi_1, \phi_2) + L(\chi_2, \phi_2, \phi_1)}{b_{\phi_1\phi_2}(R_{\phi_1\chi_1} + R_{\phi_2\chi_2} - 2s)} \\
J_2(\phi_1, \phi_2, \chi_1, \chi_2) &= -\frac{1}{4} + \frac{\mathcal{X}(\phi_1, \phi_2, \chi_1)L(\chi_1, \phi_1, \phi_2) + \mathcal{X}(\phi_2, \phi_1, \chi_2)L(\chi_2, \phi_2, \phi_1)}{16b_{\phi_1\phi_2}(R_{\phi_1\chi_1} + R_{\phi_2\chi_2} - 2s)} \\
J_3(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{1}{b_{\phi_1\phi_2}(R_{\phi_1\chi_1} + R_{\phi_2\chi_2} - 2s)}(R_{\phi_2\chi_2} - 4m_i^2s)L(\chi_2, \phi_2, \phi_1) \\
&\quad -(R_{\phi_1\chi_1} + 4m_i^2s - 2sl_i)L(\chi_1, \phi_1, \phi_2) \\
J_4(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{1}{b_{\phi_1\phi_2}(R_{\phi_1\chi_1} + R_{\phi_2\chi_2} - 2s)}((\mathcal{C}(\chi_1, \phi_1, \phi_2) - 4ds)L(\chi_1, \phi_1, \phi_2) - \\
&\quad 2(4ds + \mathcal{B}(\chi_2, \phi_2, \phi_1))\mathcal{A}(\chi_2, \phi_2, \phi_1)) \\
J_5(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{1}{b_{\phi_1\phi_2}(R_{\phi_1\chi_1} + R_{\phi_2\chi_2} - 2s)}((4ds + \mathcal{B}(\chi_1, \phi_1, \phi_2))L(\chi_1, \phi_1, \phi_2) \\
&\quad - 2(\mathcal{C}(\chi_2, \phi_2, \phi_1) - 4ds)\mathcal{A}(\chi_2, \phi_2, \phi_1)) \\
J_6(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{m_j m_{\chi_2}}{8b_{\phi_1\phi_2}m_1^2(R_{\phi_1\chi_1} - R_{\phi_2\chi_2})}(2b_{\phi_1\phi_2}(R_{\phi_1\chi_1} - R_{\phi_2\chi_2}) - 8m_i^2m_1^2s^2 \\
&\quad \mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2) + 6m_1^2s\mathcal{Y}_2(\chi_1, \chi_2, \phi_1, \phi_2) - \mathcal{Z}_2(\chi_1, \chi_2, \phi_1, \phi_2)) \\
J_7(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{3m_j m_{\chi_1} s}{4b_{\phi_1\phi_2}(R_{\phi_1\chi_1} - R_{\phi_2\chi_2})}(-2l_i s + 4m_i^2s)\mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2) \\
&\quad + \mathcal{Y}_2(\chi_1, \chi_2, \phi_1, \phi_2) \\
J_8(\phi_1, \phi_2, \chi_1, \chi_2) &= -\frac{3m_i m_{\chi_2} s}{4b_{\phi_1\phi_2}(R_{\phi_1\chi_1} - R_{\phi_2\chi_2})}((4ds - 2l_{\phi_1\phi_2}s)\mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2) \\
&\quad + \mathcal{Y}_2(\chi_1, \chi_2, \phi_1, \phi_2)) \\
J_9(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{m_i m_{\chi_1}}{4m_1^2} + \frac{m_i m_{\chi_1} \mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{8b_{\phi_1\phi_2}m_1^2(R_{\phi_1\chi_1} - R_{\phi_2\chi_2})}(2m_1^2s(\tau_{\phi_1\phi_2} - \tau_{\phi_2\phi_1}) \\
&\quad - 8d_{\phi_1\phi_2}(l_{\phi_1\phi_2} - m_1^2)s^2) + \frac{m_i m_{\chi_1} \mathcal{Y}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{8b_{\phi_1\phi_2}m_1^2(R_{\phi_1\chi_1} - R_{\phi_2\chi_2})} \\
&\quad (4d_{\phi_1\phi_2}s + 2l_{\phi_1\phi_2}s - 2m_1^2s) - \frac{m_i m_{\chi_1} \mathcal{Z}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{8b_{\phi_1\phi_2}m_1^2(R_{\phi_1\chi_1} - R_{\phi_2\chi_2})}
\end{aligned}$$

$$\begin{aligned}
J_{10}(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{1}{2} + \frac{\mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{4 b_{\phi_1 \phi_2} m_1^2 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (-b_{\phi_1 \phi_2} + s^2 (4 \tau_{\phi_1 \phi_2} + 3 \omega_{\phi_1 \phi_2} \\
&\quad - \omega_{\phi_2 \phi_1}) m_i^2 m_1^2 + (4 d_{\phi_1 \phi_2} l_{\phi_1 \phi_2} m_i^2 + (-3 \tau_{\phi_1 \phi_2} + 4 d (d_{\phi_1 \phi_2} + 2 l_i \\
&\quad - 3 m_i^2)) m_1^2) s) - \frac{\mathcal{Y}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{32 b_{\phi_1 \phi_2} m_1^2 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2}) s} (2 m_1^2 s (-7 \tau_{\phi_1 \phi_2} \\
&\quad - 3 (\tau_{\phi_2 \phi_1} + \omega_{\phi_1 \phi_2}) + \omega_{\phi_2 \phi_1}) \\
&\quad + 16 (d + m_i^2) (d_{\phi_1 \phi_2} + 2 m_1^2) s^2) + \frac{\mathcal{Z}_2(\chi_1, \chi_2, \phi_1, \phi_2) (4 s - 8 m_1^2 s)}{32 b_{\phi_1 \phi_2} m_1^2 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2}) s} \\
J_{11}(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{m_{\chi_1} m_{\chi_2}}{4 m_1^2} + \frac{d s^2 m_{\chi_1} m_{\chi_2} \mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} \\
&\quad + \frac{l_{\phi_1 \phi_2} s m_{\chi_1} m_{\chi_2} \mathcal{Y}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{4 b_{\phi_1 \phi_2} m_1^2 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} - \frac{m_{\chi_1} m_{\chi_2} \mathcal{Z}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{8 b_{\phi_1 \phi_2} m_1^2 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} \\
J_{12}(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{m_i m_j}{4 m_1^2} + \frac{s^2 m_i m_j \mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{2 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (-6 d_{\phi_1 \phi_2} + l_i - 2 m_i^2) \\
&\quad + \frac{s m_i m_j \mathcal{Y}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{2 b_{\phi_1 \phi_2} m_1^2 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (d_{\phi_1 \phi_2} + m_1^2) \\
&\quad - \frac{m_i m_j \mathcal{Z}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{8 b_{\phi_1 \phi_2} m_1^2 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} \\
J_{13}(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{3 m_i m_j m_{\chi_1} m_{\chi_2} s^2 \mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} \\
J_{14}(\phi_1, \phi_2, \chi_1, \chi_2) &= -\frac{b_{\phi_1 \phi_2}^2}{48 m_1^4 s^2} - \frac{1}{16 m_1^4 s^2} (R_{\phi_1 \chi_1}^2 + R_{\phi_2 \chi_2}^2 + R_{\phi_1 \chi_1} R_{\phi_2 \chi_2} - 3 l_i s \\
&\quad (R_{\phi_1 \chi_1} + R_{\phi_2 \chi_2})) - \frac{1}{16 m_1^4} (-8 d d_{\phi_1 \phi_2} + 2 l_i^2 + l_i l_{ia} + 8 d_{\phi_1 \phi_2} m_i^2 \\
&\quad - 4 m_1^2 (l_{ia} + m_1^2 + 2 s)) + \frac{s^2 \mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{8 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (-2 l_i^2 m_1^2 s \\
&\quad + 4 d_{\phi_1 \phi_2} m_i^2 (-4 l_{ia} m_1^2 + l_i s) - m_1^4 (-12 l_{ia} m_i^2 + 2 l_i s)) \\
&\quad - \frac{d s^2 \mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (-2 d_{\phi_1 \phi_2}^2 m_i^2 + d_{\phi_1 \phi_2} l_i m_1^2 + (5 d_{\phi_1 \phi_2} \\
&\quad - 4 l_i + 5 m_i^2) m_1^4) - \frac{l_i s \mathcal{Y}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{16 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} \\
&\quad (-8 d d_{\phi_1 \phi_2} + l_i^2 + l_i l_{ia} + 8 d_{\phi_1 \phi_2} m_i^2 - 4 m_1^2 (l_{ia} + m_1^2 + 2 s)) \\
&\quad + \frac{\mathcal{Z}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{16 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (-4 d d_{\phi_1 \phi_2} + 2 l_i^2 + 4 d_{\phi_1 \phi_2} m_i^2 + l_i s \\
&\quad - 2 m_1^2 (l_{ia} + m_1^2 + 2 s)) - \frac{4 l_i \mathcal{R}_1(\chi_1, \chi_2, \phi_1, \phi_2) - \mathcal{Q}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{32 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2}) s^2}
\end{aligned}$$

$$\begin{aligned}
J_{15}(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{m_{\chi_1} m_{\chi_2} (m_1^2 - d_{\phi_1 \phi_2})}{2 m_1^2} - \frac{m_{\chi_1} m_{\chi_2} s^2 \mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{4 b_{\phi_1 \phi_2} m_1^2 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (l_i (l_i - l_{ia}) \\
&\quad m_1^2 + 12 d m_1^4 + d_{\phi_1 \phi_2} (8 d d_{\phi_1 \phi_2} - 2 l_i s)) \\
&\quad - \frac{l_i m_{\chi_1} m_{\chi_2} s \mathcal{Y}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{2 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (d_{\phi_1 \phi_2} - m_1^2) \\
&\quad + \frac{m_{\chi_1} m_{\chi_2} \mathcal{Z}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{4 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (d_{\phi_1 \phi_2} - m_1^2) \\
J_{16}(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{m_i m_j (-d_{\phi_1 \phi_2} + m_1^2)}{2 m_1^4} + \frac{m_i m_j s^2 \mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{2 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (4 d_{\phi_1 \phi_2}^2 m_i^2 \\
&\quad - l_i^2 m_1^2 - (6 d_{\phi_1 \phi_2} + l_i - 6 m_i^2) m_1^4 + 4 d_{\phi_1 \phi_2} m_1^2 s) \\
&\quad + \frac{m_i m_j (d_{\phi_1 \phi_2} - m_1^2)}{4 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (-2 l_i s \mathcal{Y}_2(\chi_1, \chi_2, \phi_1, \phi_2) \\
&\quad + \mathcal{Z}_2(\chi_1, \chi_2, \phi_1, \phi_2)) \\
J_{17}(\phi_1, \phi_2, \chi_1, \chi_2) &= -\frac{m_i m_j m_{c j i 1} m_{\chi_2} s^2 \mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (-2 d_{\phi_1 \phi_2}^2 + 5 m_1^4) \\
J_{18}(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{m_j m_{\chi_2} (2 d_{\phi_1 \phi_2} + m_1^2)}{4 m_1^4} + \frac{m_j m_{\chi_2} s^2 \mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} \\
&\quad (-2 d_{\phi_1 \phi_2}^2 m_i^2 + (d_{\phi_1 \phi_2} - l_i) l_i m_1^2 + 3 m_i^2 m_1^4) \\
&\quad + \frac{m_j m_{\chi_2} s \mathcal{Y}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{4 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (2 d_{\phi_1 \phi_2} l_i - 2 (d_{\phi_1 \phi_2} - 2 l_i) m_1^2 - 5 m_1^4) \\
&\quad - \frac{m_j m_{\chi_2} (2 d_{\phi_1 \phi_2} + m_1^2)}{8 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} \mathcal{Z}_2(\chi_1, \chi_2, \phi_1, \phi_2) \\
J_{19}(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{m_j m_{\chi_1} (2 d_{\phi_1 \phi_2} + m_1^2)}{4 m_1^4} + \frac{m_j m_{\chi_1} s^2 \mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{2 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} \\
&\quad (-4 d_{\phi_1 \phi_2}^2 m_i^2 + l_i^2 m_1^2 + (-5 l_i + 6 m_i^2) m_1^4) \\
&\quad + \frac{m_j m_{\chi_1} s \mathcal{Y}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{4 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (-2 l_i m_1^2 + 5 m_1^4 \\
&\quad + 2 d_{\phi_1 \phi_2} (l_i + m_1^2)) - \frac{m_j m_{\chi_1} (2 d_{\phi_1 \phi_2} + m_1^2)}{8 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} \mathcal{Z}_2(\chi_1, \chi_2, \phi_1, \phi_2) \\
J_{20}(\phi_1, \phi_2, \chi_1, \chi_2) &= \frac{m_i m_{\chi_2} (2 d_{\phi_1 \phi_2} + m_1^2)}{4 m_1^4} - \frac{m_i m_{\chi_2} s^2 \mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{4 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} ((4 d_{\phi_1 \phi_2} - l_i) \\
&\quad (l_i - l_{ia}) m_1^2 + d (-8 d_{\phi_1 \phi_2}^2 + 12 m_1^4) + 2 (d_{\phi_1 \phi_2} l_i + m_1^4) s) \\
&\quad + \frac{m_i m_{\chi_2} s \mathcal{Y}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{8 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (4 d_{\phi_1 \phi_2} l_i + (8 d_{\phi_1 \phi_2} - l_i + 3 l_{ia}) m_1^2 \\
&\quad + 2 m_1^4) - \frac{m_i m_{\chi_2} (2 d_{\phi_1 \phi_2} + m_1^2)}{8 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} \mathcal{Z}_2(\chi_1, \chi_2, \phi_1, \phi_2)
\end{aligned}$$

$$\begin{aligned}
J_{21}(\phi_1, \phi_2, \chi_1, \chi_2) = & \frac{m_i m_{\chi_1} (2 d_{\phi_1 \phi_2} + m_1^2)}{4 m_1^4} + \frac{m_i m_{\chi_1} s^2 \mathcal{X}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{4 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (-m_1^2 (l_i - l_{ia}) \\
& (2 l_i - m_1^2) + 4 d (2 d_{\phi_1 \phi_2}^2 - 3 m_1^4) - 2 d_{\phi_1 \phi_2} (l_i - 4 m_1^2) s) \\
& \frac{m_i m_{\chi_1} s \mathcal{Y}_2(\chi_1, \chi_2, \phi_1, \phi_2)}{8 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} (4 d_{\phi_1 \phi_2} l_i + (-8 d_{\phi_1 \phi_2} + 5 l_i - 3 l_{ia}) m_1^2 \\
& - 2 m_1^4) - \frac{m_i m_{\chi_1} (2 d_{\phi_1 \phi_2} + m_1^2)}{8 b_{\phi_1 \phi_2} m_1^4 (R_{\phi_1 \chi_1} - R_{\phi_2 \chi_2})} \mathcal{Z}_2(\chi_1, \chi_2, \phi_1, \phi_2)
\end{aligned}$$

$$\begin{aligned}
\mathcal{B}_1(\chi, \phi_1, \phi_2) = & -\frac{b_{\phi_1 \phi_2} m_\chi^4}{6 s^2} + \frac{8}{s^2} (s - m_\chi^2)^2 (m_1^2 - m_2^2)^2 \tau_{\phi_1 \phi_2} \tau_{\phi_2 \phi_1} - \frac{d}{6} ((m_1^2 + m_2^2 \\
& - 2 d_{\phi_1 \phi_2}) m_\chi^4 + (s - 2 m_\chi^2) (m_1^2 - m_2^2)^2)
\end{aligned}$$

$$\mathcal{B}_2(\chi, \phi_1, \phi_2) = m_i m_j ((s - 2 m_\chi^2) (m_1^2 - m_2^2)^2 + (m_i^2 + m_j^2 - 2 d_{\phi_1 \phi_2}) m_\chi^4)$$

$$\begin{aligned}
\mathcal{B}_3(\phi, \chi, \phi_1, \phi_2) = & b_{\phi_1 \phi_2} m_\phi^2 + (m_\phi^2 (R_{\phi_1 \chi} - 2 s l_i) - 2 s l_i (m_1^2 - m_2^2)) \\
& \mathcal{A}(\chi, \phi_1, \phi_2)
\end{aligned}$$

$$\begin{aligned}
\mathcal{B}_4(\phi, \chi, \phi_1, \phi_2) = & 2 b_{\phi_1 \phi_2} m_\phi^2 + (2 (m_1^2 - m_2^2) l_{ia} + m_\phi^2 (\mathcal{C}(\chi, \phi_1, \phi_2) - 2 s l_{\phi_1 \phi_2})) \\
& \mathcal{A}(\chi, \phi_1, \phi_2)
\end{aligned}$$

$$\begin{aligned}
\mathcal{B}_5(\phi, \chi, \phi_1, \phi_2) = & \frac{R_{\phi_1 \chi}^2}{8 b_{\phi_1 \phi_2} s} L(\chi, \phi_1, \phi_2) + \frac{R_{\phi_1 \chi}}{8 b_{\phi_1 \phi_2} m_\phi^2 s} (-2 b_{\phi_1 \phi_2} m_\phi^2 - L(\chi, \phi_1, \phi_2) \\
& (l_{\phi_2 \phi_1} m_\phi^2 (m_i^2 - m_j^2) + (2 m_i^2 m_\phi^2 - (m_1^2 - m_2^2) (m_i^2 - m_j^2)) s \\
& + l_{\phi_2 \phi_1} m_\phi^2 (2 \tau_{\phi_1 \phi_2} + 3 s))) + \frac{1}{4 m_\phi^2} ((\tau_{\phi_1 \phi_2} + 2 m_i^2) m_\phi^2 + (m_1^2 - m_2^2) \\
& (m_i^2 - m_j^2)) - \frac{s^2 L(\chi, \phi_1, \phi_2)}{4 b_{\phi_1 \phi_2} m_\phi^2 s} ((m_1^2 - m_2^2) m_i^2 ((m_1^2 - m_2^2) + m_\phi^2 \\
& + (m_i^2 - m_j^2)) + m_\phi^2 (2 d(-d_{\phi_1 \phi_2} + m_1^2) + 2 l_{\phi_1 \phi_2} (l_i + m_i^2) + m_i^2 \\
& (m_i^2 - m_j^2)) + \frac{l_{ia}}{4 s} (2 (m_1^2 - m_2^2) + s)
\end{aligned}$$

$$\begin{aligned}
\mathcal{B}_6(\phi, \chi, \phi_1, \phi_2) = & b_{\phi_1 \phi_2} m_\phi^2 + (2 (m_1^2 - m_2^2) (l_{\phi_1 \phi_2} - l_i) s + m_\phi^2 (R_{\phi_1 \chi} \\
& + 2 (d_{\phi_1 \phi_2} - m_1^2 - l_i) s)) \mathcal{A}(\chi, \phi_1, \phi_2)
\end{aligned}$$

$$\begin{aligned}
\mathcal{B}_7(\chi, \phi_1, \phi_2) = & -\frac{1}{48 m_\chi^4 s^2} (b_{\phi_1 \phi_2}^2 m_\chi^2 + 24 d (m_\chi^2 - s)^2 s^2 + 12 l_i l_{ia} (2 m_\chi^2 - s) s^2) \\
& - \frac{l_{\phi_1 \phi_2}^2}{48 m_\chi^6 s^2} (l_i l_{ia} (m_\chi^2 - s)^2 + 2 d (2 m_\chi^2 - s) s^2)
\end{aligned}$$

$$\mathcal{B}_8(\chi, \phi_1, \phi_2) = \frac{1}{8 m_\chi^6} (l_{\phi_1 \phi_2}^2 (-2 m_\chi^2 + s) - 4 m_\chi^2 (3 m_\chi^4 - 2 m_\chi^2 s + s^2))$$

$$\begin{aligned}
\mathcal{B}_9(\phi, \chi, \phi_1, \phi_2) = & -\frac{m_j}{8 m_\phi^4 s} (l_{\phi_1 \phi_2} l_i (m_\phi^2 - s) + m_\phi^2 (R_{\phi_1 \chi} + 2 s (-3 m_\phi^2 + s))) \\
& + \frac{m_j L(\chi, \phi_1, \phi_2)}{16 b_{\phi_1 \phi_2} m_\phi^4 s} (2 m_\phi^4 s (-3 R_{\phi_1 \chi} + 4 m_i^2 s) + l_{\phi_1 \phi_2} s (-l_i R_{\phi_1 \chi} \\
& + 2 l_{\phi_1 \phi_2} m_i^2 s) + m_\phi^2 (R_{\phi_1 \chi}^2 + 2 (-\tau_{\phi_1 \phi_2} + 2 l_i^2 + R_{\phi_1 \chi}) s^2 - 8 m_i^2 s^3))
\end{aligned}$$

$$\begin{aligned}
\mathcal{B}_{10}(\phi, \chi, \phi_1, \phi_2) &= \frac{m_i}{16 m_\phi^4} (\tau_{\phi_1 \phi_2} - \omega_{\phi_1 \phi_2} + 12 m_\phi^4 - 4 m_\phi^2 s) + \frac{m_i L(\chi, \phi_1, \phi_2)}{32 b_{\phi_1 \phi_2} m_\phi^4 s} \\
&\quad (\tau_{\phi_1 \phi_2} - \omega_{\phi_1 \phi_2} (12 m_\phi^4 - R_{\phi_1 \chi}) - 4 \tau_{\phi_1 \phi_2} m_\phi^2 s + l_{\phi_1 \phi_2}^2 (l_i^2 - l_i l_{ia} \\
&\quad + 4 d s) + 4 m_\phi^2 (3 m_\phi^2 - s) (-R_{\phi_1 \chi} + 4 d s)) \\
\mathcal{B}_{11}(\phi, \chi, \phi_1, \phi_2) &= \frac{m_\chi}{8 m_\phi^4 s} (m_\phi^2 R_{\phi_1 \chi} - l_{\phi_1 \phi_2} (l_i m_\phi^2 + s (-2 m_\phi^2 + s))) + \frac{m_\chi L(\chi, \phi_1, \phi_2)}{16 b_{\phi_1 \phi_2} m_\phi^4 s} \\
&\quad (l_{\phi_1 \phi_2}^2 s (l_i l_{ia} - (2 d + l_i) s) + l_{\phi_1 \phi_2} R_{\phi_1 \chi} (2 l_i m_\phi^2 + s (-2 m_\phi^2 + s)) \\
&\quad - m_\phi^2 (R_{\phi_1 \chi}^2 + 4 s^2 (l_i l_{ia} + 2 d m_\phi^2 - 2 d s))) \\
\mathcal{B}_{12}(\phi, \chi, \phi_1, \phi_2) &= -\frac{m_i m_j m_\chi s L(\chi, \phi_1, \phi_2)}{8 b_{\phi_1 \phi_2} m_\phi^4} (l_{\phi_1 \phi_2}^2 + 12 m_\phi^4 - 4 m_\phi^2 s) \\
\mathcal{B}_{13}(\phi, \chi, \phi_1, \phi_2) &= \frac{m_\chi L(\chi, \phi_1, \phi_2)}{16 b_{\phi_1 \phi_2} m_\phi^4} (l_{\phi_2 \phi_1} \rho(\chi, \phi_1, \phi_2) (-2 m_\phi^2 + s) + l_{\phi_2 \phi_1}^2 (l_i l_{ia} - 2 d s) \\
&\quad - 4 m_\phi^2 s (l_i l_{ia} + 2 d m_\phi^2 - 2 d s)) + \frac{m_\chi l_{\phi_2 \phi_1}}{4 b_{\phi_1 \phi_2} m_\phi^2} \\
\mathcal{B}_{14}(\phi, \chi, \phi_1, \phi_2) &= -\frac{m_i m_j m_\chi s L(\chi, \phi_1, \phi_2)}{8 b_{\phi_1 \phi_2} m_\phi^4} (l_{\phi_2 \phi_1}^2 + 12 m_\phi^4 - 4 m_\phi^2 s) \\
\mathcal{B}_{15}(\phi, \chi, \phi_1, \phi_2) &= \frac{m_j L(\chi, \phi_1, \phi_2)}{32 b_{\phi_1 \phi_2} m_\phi^4} (l_{\phi_2 \phi_1} (-l_{\phi_1 \phi_2} \rho(\chi, \phi_1, \phi_2) + 4 d_{\phi_1 \phi_2} l_i s) \\
&\quad + l_{\phi_2 \phi_1}^2 (\rho(\chi, \phi_1, \phi_2) - 4 m_i^2 s) + 4 m_\phi^2 (3 m_\phi^2 - s) (R_{\phi_1 \chi} - 4 m_i^2 s)) \\
&\quad - \frac{m_j}{2 m_\phi^4} (l_{\phi_2 \phi_1}^2 - l_{\phi_2 \phi_1} l_{\phi_1 \phi_2} + 12 m_\phi^4 - 4 m_\phi^2 s) \\
\mathcal{B}_{16}(\phi, \chi, \phi_1, \phi_2) &= \frac{m_i L(\chi, \phi_1, \phi_2)}{32 b_{\phi_1 \phi_2} m_\phi^4} (2 l_{\phi_2 \phi_1}^2 l_i l_{ia} + 8 d_{\phi_1 \phi_2} l_{\phi_2 \phi_1} l_{ia} m_\phi^2 + (l_{\phi_2 \phi_1}^2 \\
&\quad - l_{\phi_2 \phi_1} l_{\phi_1 \phi_2}) \rho(\chi, \phi_1, \phi_2) - 4 m_\phi^2 (l_{\phi_2 \phi_1} - m_1^2) R_{\phi_1 \chi} - 2 (l_{\phi_2 \phi_1}^3 \\
&\quad + 4 l_i l_{ia} m_\phi^2 + 8 d m_\phi^4 + 12 l_{\phi_2 \phi_1} m_\phi^4 - 4 l_{\phi_1 \phi_2} m_\phi^4 - l_{\phi_2 \phi_1} R_{\phi_1 \chi}) s \\
&\quad + 8 (2 d + 2 s) m_\phi^2 s^2) + \frac{m_i}{32 m_\phi^4} (2 l_{\phi_2 \phi_1} (-l_{\phi_2 \phi_1} + l_{\phi_1 \phi_2}) + 8 m_\phi^2 (m_\phi^2 + s)) \\
\mathcal{B}_{17}(\phi, \chi, \phi_1, \phi_2) &= s (m_2^4 + (m_1^2 - s)^2 - 2 m_2^2 (m_1^2 + s)) \frac{\mathcal{A}(\chi, \phi_1, \phi_2)}{4 b_{\phi_1 \phi_2} m_1^2 (m_2^2 - s) s} \\
\mathcal{B}_{18}(\phi, \chi, \phi_1, \phi_2) &= (6 m_\phi^2 - s) s - (m_1^2 - m_2^2) (2 m_\phi^2 + s) \\
\mathcal{B}_{19}(\phi, \chi, \phi_1, \phi_2) &= 4 d_{\phi_1 \phi_2} s^2 - 3 m_\phi^2 (m_2^2 - m_1^2 + 2 s^2) \\
\mathcal{B}_{20}(\phi, \chi, \phi_1, \phi_2) &= \frac{m_j}{8 m_1^4} (d_{\phi_1 \phi_2} l_i + 2 (-d_{\phi_1 \phi_2} + l_i) m_1^2 + 4 m_1^4) + \frac{s m_j L(\chi, \phi_1, \phi_2)}{4 b_{\phi_1 \phi_2} m_1^2} \\
&\quad (2 d_{\phi_1 \phi_2}^2 m_i^2 + l_i (-d_{\phi_1 \phi_2} + l_i) m_1^2) - \frac{m_j R_{\phi_1 \chi}}{8 b_{\phi_1 \phi_2} m_1^4 s} (b_{\phi_1 \phi_2} (d_{\phi_1 \phi_2} + 2 m_1^2) \\
&\quad + (d_{\phi_1 \phi_2} (l_i - m_1^2) + 2 m_1^2 (l_i + m_1^2)) s L(\chi, \phi_1, \phi_2))
\end{aligned}$$

$$\begin{aligned}
\mathcal{B}_{21}(\phi, \chi, \phi_1, \phi_2) &= \frac{m_i}{8 b_{\phi_1 \phi_2} m_1^4} (d_{\phi_1 \phi_2} l_i + 2(-d_{\phi_1 \phi_2} + l_i) m_1^2 + 4 m_1^4) + \frac{m_i s L(\chi, \phi_1, \phi_2)}{8 b_{\phi_1 \phi_2} m_1^4} \\
&\quad (-4 d d_{\phi_1 \phi_2}^2 - (d_{\phi_1 \phi_2} - l_i)(l_i - l_{ia}) m_1^2 + (d_{\phi_1 \phi_2} l_i + 4 m_1^4) s) \\
&\quad - \frac{m_i R_{\phi_1 \chi}}{8 b_{\phi_1 \phi_2} s m_1^4} (b_{\phi_1 \phi_2} (d_{\phi_1 \phi_2} + 2 m_1^2) + (d_{\phi_1 \phi_2} (l_i - m_1^2) + 2 m_1^2 (l_i + m_1^2)) \\
&\quad s L(\chi, \phi_1, \phi_2)) \\
\mathcal{B}_{22}(\phi, \chi, \phi_1, \phi_2) &= \frac{m_\chi s}{8 b_{\phi_1 \phi_2} m_1^4} (-2 b_{\phi_1 \phi_2} d_{\phi_1 \phi_2} + (4 d (d_{\phi_1 \phi_2}^2 + 2 m_1^4) + d_{\phi_1 \phi_2} (R_{\phi_1 \chi} - l_i s)) \\
&\quad L(\chi, \phi_1, \phi_2)) \\
\mathcal{B}_{23}(\phi, \chi, \phi_1, \phi_2) &= -\frac{m_i m_j m_\chi s L(\chi, \phi_1, \phi_2)}{2 b_{\phi_1 \phi_2} m_1^4} (d_{\phi_1 \phi_2}^2 + 2 m_1^4)
\end{aligned}$$

$$\begin{aligned}
\mathcal{B}_{24}(\phi, \chi, \phi_1, \phi_2) &= \frac{1}{192 b_{\phi_1 \phi_2} m_1^4 m_\phi^2 s^2} (2 b_{\phi_1 \phi_2} (2 b_{\phi_1 \phi_2}^2 m_\phi^2 (2 m_1^2 - s) + 24 d_{\phi_1 \phi_2} (d - m_i^2) \\
&\quad m_\phi^2 s^3 - 12 m_1^4 s^2 (l_i^2 - 3 l_i l_{ia} + 4 d s + 2 m_\phi^2 (-2 (2 d + l_i + m_i^2) + 5 s)) \\
&\quad + 6 m_1^2 s^2 ((-2 d_{\phi_1 \phi_2} + s) (l_i^2 - 3 l_i l_{ia} + 4 d s) + m_\phi^2 (-l_i^2 + l_i (7 l_{ia} - 2 s) \\
&\quad + 2 s (4 d_{\phi_1 \phi_2} + 2 m_i^2 + s)))) + 3 s^2 (-m_\phi^2 (l_i^3 + 8 d_{\phi_1 \phi_2} l_i m_i^2 - 8 d_{\phi_1 \phi_2} l_{ia} m_i^2 \\
&\quad + l_i^2 (l_{ia} - 2 s)) s^2 + 16 m_1^6 s (l_i l_{ia} + 6 d m_\phi^2 - 2 d s) + 4 m_1^4 s (-l_i^3 + 3 l_i^2 l_{ia} \\
&\quad - 2 l_i (2 d + l_{ia}) s + 4 d s^2 + 2 m_\phi^2 (2 (l_i - 4 l_{ia}) m_i^2 - l_i s + 2 d (-6 d_{\phi_1 \phi_2} + 6 l_i \\
&\quad + s))) + 2 m_1^2 ((-2 d_{\phi_1 \phi_2} + s) s (l_i (l_i^2 + 4 d_{\phi_1 \phi_2} l_{ia} - 3 l_i l_{ia}) + 4 d (-2 d_{\phi_1 \phi_2} \\
&\quad + l_i) s) + m_\phi s (4 d_{\phi_1 \phi_2} (l_i^2 - l_i l_{ia} + 4 l_{ia} m_i^2) - 8 d d_{\phi_1 \phi_2} (2 l_i + s) + l_i (-l_i^2 \\
&\quad + 7 l_i l_{ia} - 2 l_i s + 4 m_i^2 s + 2 s^2)))) L(\chi, \phi_1, \phi_2)) + \frac{R_{\phi_1 \chi}}{64 b_{\phi_1 \phi_2} m_1^2 m_\phi^2 s} \\
&\quad (4 b_{\phi_1 \phi_2} m_\phi^2 (-4 m_1^2 (-d_{\phi_1 \phi_2} + l_i + 2 m_1^2) + 2 (l_i + m_1^2) s - s^2) - s \\
&\quad (2 m_1^2 (-2 l_i (l_i - 3 l_{ia}) (d_{\phi_1 \phi_2} + m_1^2) + (3 l_i^2 + 4 d_{\phi_1 \phi_2} l_{ia} + 7 l_i l_{ia} \\
&\quad + 8 (2 d + 2 l_i + m_i^2) m_1^2) m_\phi^2) + 4 (2 d m_1^2 + (l_i + m_1^2) m_\phi^2) s^2 - s (2 m_1^2 (-l_i \\
&\quad (l_i - 3 l_{ia}) + 8 d (d_{\phi_1 \phi_2} + m_1^2)) + m_\phi^2 (-8 d d_{\phi_1 \phi_2} + 8 d_{\phi_1 \phi_2} m_i^2 + 8 (l_i - m_i^2) \\
&\quad m_1^2 + 40 m_1^4 + l_i (4 l_i + 2 s)))) L(\chi, \phi_1, \phi_2)) + \frac{R_{\phi_1 \chi}^2}{32 b_{\phi_1 \phi_2} m_1^2 s} (b_{\phi_1 \phi_2} \\
&\quad (4 m_1^2 - 2 s) + s (-4 d_{\phi_1 \phi_2} m_1^2 + 6 l_i m_1^2 + 8 m_1^4 - 3 l_i s - 2 m_1^2 s + s^2) \\
&\quad L(\chi, \phi_1, \phi_2)) + \frac{6 R_{\phi_1 \chi}^3 L(\chi, \phi_1, \phi_2)}{192 b_{\phi_1 \phi_2} m_1^4 s^2} (s - 2 m_1^2) \\
\mathcal{B}_{25}(\phi, \chi, \phi_1, \phi_2) &= \frac{m_i m_j}{8 b_{\phi_1 \phi_2} m_1^2 m_\phi^2} (2 d_{\phi_1 \phi_2} (2 m_\phi^2 - s) + 2 m_1^2 (5 m_\phi^2 - s) + s (3 m_\phi^2 + s)) \\
&\quad (2 b_{\phi_1 \phi_2} - (R_{\phi_1 \chi} + (2 d_{\phi_1 \phi_2} - l_i - 2 m_1^2) s) L(\chi, \phi_1, \phi_2))
\end{aligned}$$

$$\begin{aligned}
\mathcal{B}_{26}(\phi, \chi, \phi_1, \phi_2) &= \frac{m_\chi m_j}{8 b_{\phi_1 \phi_2} m_1^4} (2 b_{\phi_1 \phi_2} (-4 m_1^4 + d_{\phi_1 \phi_2} (2 m_1^2 + s)) - (l_i m_1^2 s^2 - 2 m_1^4 (2 R_{\phi_1 \chi} \\
&\quad + 5 l_i s) + d_{\phi_1 \phi_2} (R_{\phi_1 \chi} s + 2 m_1^2 (R_{\phi_1 \chi} + 2 l_i s))) L(\chi, \phi_1, \phi_2))
\end{aligned}$$



$$\begin{aligned}\mathcal{B}_{27}(\phi, \chi, \phi_1, \phi_2) &= \frac{m_\chi m_i}{8 b_{\phi_1 \phi_2} m_1^4} \left( -2 b_{\phi_1 \phi_2} (-4 m_1^4 + d_{\phi_1 \phi_2} (2 m_1^2 + s)) + (l_{ia} m_1^2 s^2 - 2 m_1^4 (2 R_{\phi_1 \chi} \right. \\ &\quad \left. - 2 l_i s + 7 l_{ia} s) + d_{\phi_1 \phi_2} (s (R_{\phi_1 \chi} - l_i s + l_{ia} s) + 2 m_1^2 (R_{\phi_1 \chi} - l_i s + 3 l_{ia} s)) \right) \\ &\quad L(\chi, \phi_1, \phi_2))\end{aligned}$$

$$\begin{aligned}\mathcal{C}_1(\chi_1, \chi_2, \phi_1, \phi_2) &= \frac{1}{8 b_{\phi_1 \phi_2} m_1^2 (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} \left( (\omega_{\phi_1 \phi_2} - \tau_{\phi_1 \phi_2}) (m_i^2 - 2 m_1^2) + 4 (2 d + m_i^2) \right. \\ &\quad \left. m_1^2 s \right) \mathcal{X}_1(\chi_1, \chi_2, \phi_1, \phi_2) + \frac{s \mathcal{Y}_1(\chi_1, \chi_2, \phi_1, \phi_2)}{2 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} \left( (\tau_{\phi_1 \phi_2} - \omega_{\phi_1 \phi_2}) m_i^2 \right. \\ &\quad \left. + 2 d (m_i^2 - m_1^2) s \right) + \frac{2 d - m_i^2 + 2 m_1^2}{8 b_{\phi_1 \phi_2} m_1^2 (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} (2 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_2} - R_{\phi_1 \chi_1}) \\ &\quad + \mathcal{Z}_1(\chi_1, \chi_2, \phi_1, \phi_2))\end{aligned}$$

$$\begin{aligned}\mathcal{C}_2(\chi_1, \chi_2, \phi_1, \phi_2) &= \frac{m_{\chi_1} m_{\chi_2} (\tau_{\phi_1 \phi_2} - \omega_{\phi_1 \phi_2})}{8 b_{\phi_1 \phi_2} m_1^2 (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} \mathcal{X}_1(\chi_1, \chi_2, \phi_1, \phi_2) \\ &\quad + \frac{s^2 d m_{\chi_1} m_{\chi_2} \mathcal{Y}_1(\chi_1, \chi_2, \phi_1, \phi_2)}{b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} + \frac{m_{\chi_1} m_{\chi_2}}{8 b_{\phi_1 \phi_2} m_1^2 (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} \\ &\quad (2 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_2} - R_{\phi_1 \chi_1}) + \mathcal{Z}_1(\chi_1, \chi_2, \phi_1, \phi_2))\end{aligned}$$

$$\mathcal{C}_3(\chi_1, \chi_2, \phi_1, \phi_2) = \frac{\mathcal{Y}_1(\chi_1, \chi_2, \phi_1, \phi_2)}{b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})}$$

$$\begin{aligned}\mathcal{C}_4(\chi_1, \chi_2, \phi_1, \phi_2) &= \frac{m_j}{8 b_{\phi_1 \phi_2} m_1^2 (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} (2 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_2} - R_{\phi_1 \chi_1}) \\ &\quad + \mathcal{Z}_1(\chi_1, \chi_2, \phi_1, \phi_2))\end{aligned}$$

$$\begin{aligned}\mathcal{C}_5(\chi_1, \chi_2, \phi_1, \phi_2) &= \frac{3 m_i s}{4 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} \left[ ((\omega_{\phi_1 \phi_2} - \tau_{\phi_1 \phi_2}) - R_{\phi_1 \chi_1} + 4 d s) \right. \\ &\quad \left. L(\chi_1, \phi_1, \phi_2) - ((\omega_{\phi_1 \phi_2} - \tau_{\phi_1 \phi_2}) - R_{\phi_1 \chi_2} + 4 d s) \right. \\ &\quad \left. L(\chi_2, \phi_1, \phi_2) \right]\end{aligned}$$

$$\begin{aligned}\mathcal{C}_6(\chi_1, \chi_2, \phi_1, \phi_2) &= \frac{3 m_j s}{4 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} \left[ (3 R_{\phi_1 \chi_1} - 4 m_i^2 s) L(\chi_1, \phi_1, \phi_2) \right. \\ &\quad \left. - (3 R_{\phi_1 \chi_2} - 4 m_i^2 s) L(\chi_2, \phi_1, \phi_2) \right]\end{aligned}$$

$$\begin{aligned}\mathcal{C}_7(\chi_1, \chi_2, \phi_1, \phi_2) &= \frac{3 m_i m_j s}{2 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} \left[ (-R_{\phi_1 \chi_1} + 2 (m_1^2 - m_i^2) s) L(\chi_1, \phi_1, \phi_2) \right. \\ &\quad \left. - (-R_{\phi_1 \chi_2} + 2 (m_1^2 - m_i^2) s) L(\chi_2, \phi_1, \phi_2) \right]\end{aligned}$$

$$\begin{aligned}\mathcal{C}_8(\chi_1, \chi_2, \phi_1, \phi_2) &= \frac{\mathcal{X}_1(\chi_1, \chi_2, \phi_1, \phi_2)}{4 m_2^2, b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} \left( (m_1^2 - m_i^2) (-\tau_{\phi_1 \phi_2} + l_{\phi_1 \phi_2} s) + m_2^2 (\tau_{\phi_1 \phi_2} \right. \\ &\quad \left. - l_{\phi_2 \phi_1} s + 2 m_i^2 s) + d_{\phi_1 \phi_2} (\tau_{\phi_1 \phi_2} - \omega_{\phi_2 \phi_1} + 4 m_i^2 s) \right) \\ &\quad + \frac{\mathcal{Y}_1(\chi_1, \chi_2, \phi_1, \phi_2)}{8 m_2^2, b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} (8 d_{\phi_1 \phi_2} m_i^2 s (\tau_{\phi_1 \phi_2} - \omega_{\phi_2 \phi_1}) \\ &\quad + 4 m_i^2 m_2^2 s (\tau_{\phi_1 \phi_2} - \omega_{\phi_1 \phi_2}) - (m_1^2 - m_i^2) (8 d m_2^2 s^2 + \tau_{\phi_2 \phi_1} \omega_{\phi_2 \phi_1} \\ &\quad - 2 l_{\phi_2 \phi_1} s \tau_{\phi_1 \phi_2} + \tau_{\phi_1 \phi_2}^2)) - \frac{s - 2 m_1^2 + m_i^2}{8 m_2^2, b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} (2 b_{\phi_1 \phi_2} \\ &\quad (R_{\phi_1 \chi_2} - R_{\phi_1 \chi_1}) + \mathcal{Z}_1(\chi_1, \chi_2, \phi_1, \phi_2))\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_9(\chi_1, \chi_2, \phi_1, \phi_2) &= \frac{\mathcal{X}_1(\chi_1, \chi_2, \phi_1, \phi_2)}{4 m_2^2 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} (\tau_{\phi_1 \phi_2} - l_{\phi_2 \phi_1} s) \\
&+ \frac{\mathcal{Y}_1(\chi_1, \chi_2, \phi_1, \phi_2)}{8 m_2^2 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} (8 d m_2^2 s^2 + \omega_{\phi_2 \phi_1} \tau_{\phi_2 \phi_1} \\
&- 2 l_{\phi_2 \phi_1} s \tau_{\phi_1 \phi_2} + \tau_{\phi_1 \phi_2}^2) + \frac{1}{8 m_2^2 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} \\
&(2 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_2} - R_{\phi_1 \chi_1}) + \mathcal{Z}_1(\chi_1, \chi_2, \phi_1, \phi_2))
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_{10}(\chi_1, \chi_2, \phi_1, \phi_2) &= -\frac{m_i s L(\chi_1, \phi_1, \phi_2)}{4 m_2^2 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} (2 d_{\phi_1 \phi_2} (\omega_{\phi_2 \phi_1} + R_{\phi_1 \chi_1} - \tau_{\phi_1 \phi_2}) \\
&+ m_2^2 (\omega_{\phi_1 \phi_2} + R_{\phi_1 \chi_1} - \tau_{\phi_1 \phi_2})) + \frac{m_i s L(\chi_2, \phi_1, \phi_2)}{4 m_2^2 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} \\
&(2 d_{\phi_1 \phi_2} (\omega_{\phi_2 \phi_1} + R_{\phi_1 \chi_2} - \tau_{\phi_1 \phi_2}) + m_2^2 (\omega_{\phi_1 \phi_2} + R_{\phi_1 \chi_2} - \tau_{\phi_1 \phi_2}))
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_{11}(\chi_1, \chi_2, \phi_1, \phi_2) &= -\frac{m_j s L(\chi_1, \phi_1, \phi_2)}{4 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} (3 R_{\phi_1 \chi_1} - 4 m_i^2 s) \\
&+ \frac{m_j s L(\chi_1, \phi_1, \phi_2)}{4 b_{\phi_1 \phi_2} (R_{\phi_1 \chi_1} - R_{\phi_1 \chi_2})} (3 R_{\phi_1 \chi_2} - 4 m_i^2 s)
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_{12}(\chi_1, \chi_2, \phi, \phi) &= -\frac{1}{48 m_\phi^4 S^2} (b_{\phi\phi}^2 + 3 (R_{\phi\chi_1}^2 + R_{\phi\chi_1} R_{\phi\chi_2} + R_{\phi\chi_2}^2 - (2 l_i - l_{ia} + 4 m_\phi^2) \\
&(R_{\phi\chi_1} + R_{\phi\chi_2}) S - (l_i l_{ia} - 8 d_{\phi\phi} m_i^2 + 4 (2 d + 4 d_{\phi\phi} - 2 l_i + l_{ia}) \\
&m_\phi^2 - 4 m_\phi^4) S^2)) + \frac{S^2 \mathcal{X}_1(\chi_1, \chi_2, \phi, \phi)}{4 b_{\phi\phi} m_\phi^2 (R_{\phi\chi_1} - R_{\phi\chi_2})} (-(4 d_{\phi\phi} - l_i) (l_i - l_{ia}) \\
&m_i^2 + 2 (l_i + 2 m_i^2) m_j^2 m_\phi^2 + 4 d m_\phi^4) - \frac{S \mathcal{Y}_1(\chi_1, \chi_2, \phi, \phi)}{4 b_{\phi\phi} m_\phi^4 (R_{\phi\chi_1} - R_{\phi\chi_2})} (d_{\phi\phi} \\
&(l_i - l_{ia}) m_i^2 + 4 d_{\phi\phi} (-2 m_i^2 + m_j^2) m_\phi^2 + m_\phi^2 ((l_i - l_{ia}) (l_i + m_i^2) \\
&+ m_\phi^2 (-4 d + 2 m_j^2 + S))) + \frac{\mathcal{Z}_1(\chi_1, \chi_2, \phi, \phi)}{16 b_{\phi\phi} m_\phi^4 (R_{\phi\chi_1} - R_{\phi\chi_2})} (l_i^2 - l_i l_{ia} + 4 d_{\phi\phi} m_i^2 \\
&- 2 (2 d + 4 d_{\phi\phi} - 3 l_i + l_{ia}) m_\phi^2 + 2 m_\phi^4) + \frac{\mathcal{R}_1(\chi_1, \chi_2, \phi, \phi)}{32 b_{\phi\phi} m_\phi^4 S (R_{\phi\chi_1} - R_{\phi\chi_2})} \\
&(-3 l_i + l_{ia} - 4 m_\phi^2) + \frac{\mathcal{Q}_1(\chi_1, \chi_2, \phi, \phi)}{32 b_{\phi\phi} m_\phi^4 S^2 (R_{\phi\chi_1} - R_{\phi\chi_2})}
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_{13}(\chi_1, \chi_2, \phi, \phi) &= \frac{m_{\chi_k^\pm} m_{\chi_l^\pm} (d_{\phi\phi} - m_\phi^2)}{2 m_\phi^4} - \frac{m_{\chi_k^\pm} m_{\chi_l^\pm} S \mathcal{X}_1(\chi_1, \chi_2, \phi, \phi)}{4 b_{\phi\phi} m_\phi^2 (R_{\phi\chi_1} - R_{\phi\chi_2})} (l_i (-l_i + l_{ia}) \\
&+ 4 d m_\phi^2) + \frac{m_{\chi_k^\pm} m_{\chi_l^\pm} S \mathcal{Y}_1(\chi_1, \chi_2, \phi, \phi)}{4 b_{\phi\phi} m_\phi^2 (R_{\phi\chi_1} - R_{\phi\chi_2})} (d_{\phi\phi} (l_i - l_{ia}) - 2 l_i m_\phi^2) \\
&+ \frac{m_{\chi_k^\pm} m_{\chi_l^\pm} (d_{\phi\phi} - m_\phi^2)}{4 b_{\phi\phi} m_\phi^2 (R_{\phi\chi_1} - R_{\phi\chi_2})} \\
\mathcal{C}_{14}(\chi_1, \chi_2, \phi, \phi) &= \frac{9 m_i m_j S}{2 b_{\phi\phi} (R_{\phi\chi_1} - R_{\phi\chi_2})} (2 m_i^2 S \mathcal{X}_1(\chi_1, \chi_2, \phi, \phi) + 2 m_\phi^2 S \mathcal{X}_1(\chi_1, \chi_2, \phi, \phi) \\
&- \mathcal{Y}_1(\chi_1, \chi_2, \phi, \phi)) \\
\mathcal{C}_{15}(\chi_1, \chi_2, \phi, \phi) &= \frac{9 m_i m_j m_{\chi_k^\pm} m_{\chi_l^\pm} S}{b_{\phi\phi} (R_{\phi\chi_1} - R_{\phi\chi_2})} \\
\mathcal{C}_{16}(\chi_1, \chi_2, \phi, \phi) &= \frac{3 m_j}{4 m_\phi^2} + \frac{3 m_j}{8 m_\phi^2 b_{\phi\phi} (R_{\phi\chi_1} - R_{\phi\chi_2})} (-8 m_i^2 m_\phi^2 S^2 \mathcal{X}_1(\chi_1, \chi_2, \phi, \phi) \\
&+ 6 m_\phi^2 S \mathcal{Y}_1(\chi_1, \chi_2, \phi, \phi) - \mathcal{Z}_1(\chi_1, \chi_2, \phi, \phi)) \\
\mathcal{C}_{17}(\chi_1, \chi_2, \phi, \phi) &= \frac{3 m_i}{4 m_\phi^2} + \frac{3 m_i S^2 \mathcal{X}_1(\chi_1, \chi_2, \phi, \phi)}{4 b_{\phi\phi} m_\phi^2 (R_{\phi\chi_1} - R_{\phi\chi_2})} ((2 d_{\phi\phi} - l_i) (l_i - l_{ia}) \\
&- m_\phi^2 (-4 d + 2 S)) - \frac{3 m_i S \mathcal{Y}_1(\chi_1, \chi_2, \phi, \phi)}{8 b_{\phi\phi} m_\phi^2 (R_{\phi\chi_1} - R_{\phi\chi_2})} ((4 d_{\phi\phi} - 3 l_i + l_{ia} - 2 m_\phi^2) \\
&- \frac{3 m_i \mathcal{Z}_1(\chi_1, \chi_2, \phi, \phi)}{8 b_{\phi\phi} m_\phi^2 (R_{\phi\chi_1} - R_{\phi\chi_2})}
\end{aligned}$$

Where  $m_i, m_j$  are masses of  $\chi_i^0$  and  $\chi_j^0$  respectively.

## References

- [1] S. P. Martin, “A Supersymmetry primer,” *Adv.Ser.Direct.High Energy Phys.*, vol. 21, pp. 1–153, 2010.
- [2] K. Griest and D. Seckel, “Three exceptions in the calculation of relic abundances,” *Phys.Rev.*, vol. D43, pp. 3191–3203, 1991.
- [3] M. Srednicki, R. Watkins, and K. A. Olive, “Calculations of Relic Densities in the Early Universe,” *Nucl.Phys.*, vol. B310, p. 693, 1988.
- [4] J. McDonald, K. A. Olive, and M. Srednicki, “Relic densities of neutralinos,” *Phys.Lett.*, vol. B283, pp. 80–84, 1992.
- [5] S. Mizuta and M. Yamaguchi, “Coannihilation effects and relic abundance of Higgsino dominant LSP(s),” *Phys.Lett.*, vol. B298, pp. 120–126, 1993.
- [6] M. Drees and M. M. Nojiri, “The Neutralino relic density in minimal  $N = 1$  supergravity,” *Phys.Rev.*, vol. D47, pp. 376–408, 1993.
- [7] M. Drees, M. M. Nojiri, D. Roy, and Y. Yamada, “Light Higgsino dark matter,” *Phys.Rev.*, vol. D56, pp. 276–290, 1997.

- [8] J. Edsjo and P. Gondolo, “Neutralino relic density including coannihilations,” *Phys.Rev.*, vol. D56, pp. 1879–1894, 1997.
- [9] T. Nihei, L. Roszkowski, and R. Ruiz de Austri, “Exact cross-sections for the neutralino WIMP pair annihilation,” *JHEP*, vol. 0203, p. 031, 2002.
- [10] T. Nihei, L. Roszkowski, and R. Ruiz de Austri, “Exact cross-sections for the neutralino slepton coannihilation,” *JHEP*, vol. 0207, p. 024, 2002.
- [11] D. Choudhury, R. Garani, and S. K. Vempati, “Flavored Co-annihilations,” *JHEP*, vol. 1206, p. 014, 2012.
- [12] B. Herrmann, M. Klasen, and Q. Le Boulc’h, “Impact of squark flavour violation on neutralino dark matter,” *Phys.Rev.*, vol. D84, p. 095007, 2011.
- [13] J. Hisano, T. Moroi, K. Tobe, and M. Yamaguchi, “Lepton flavor violation via right-handed neutrino Yukawa couplings in supersymmetric standard model,” *Phys.Rev.*, vol. D53, pp. 2442–2459, 1996.
- [14] S. Profumo, “Neutralino dark matter, b tau Yukawa unification and nonuniversal sfermion masses,” *Phys.Rev.*, vol. D68, p. 015006, 2003.
- [15] H. Baer, A. Mustafayev, S. Profumo, A. Belyaev, and X. Tata, “Direct, indirect and collider detection of neutralino dark matter in SUSY models with non-universal Higgs masses,” *JHEP*, vol. 0507, p. 065, 2005.
- [16] R. Sundrum, “SUSY Splits, But Then Returns,” *JHEP*, vol. 1101, p. 062, 2011.
- [17] N. Arkani-Hamed, S. Dimopoulos, G. Giudice, and A. Romanino, “Aspects of split supersymmetry,” *Nucl.Phys.*, vol. B709, pp. 3–46, 2005.
- [18] J. Rosiek, “Complete Set of Feynman Rules for the Minimal Supersymmetric Extension of the Standard Model,” *Phys.Rev.*, vol. D41, p. 3464, 1990.